

Setting a Robust SLAM problem in a set of equations

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1st year PhD Student in Solving Simultaneous Localization And Mapping problems in the field of underwater robotics using set membership methods

<http://www.ensieta.fr/sliwka>

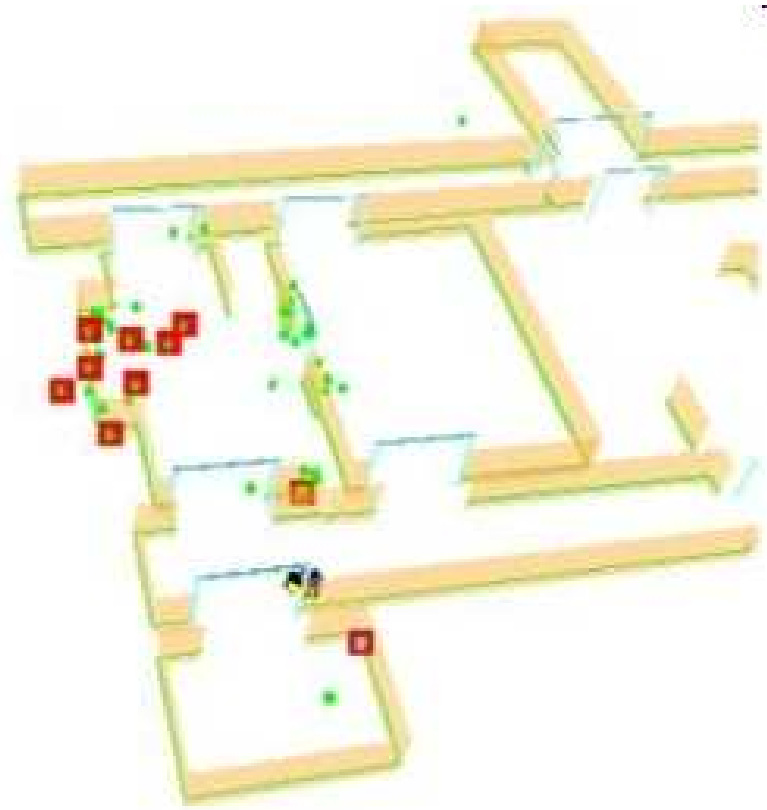
ENSIETA

2 rue François Verny

29200 Brest

SLAM

- SLAM: Simultaneous Localization And Mapping
- **non-linear** equations
- **Outliers** in the data
- Areas determined by **polygons** and **segments**



Example of a robots movement area

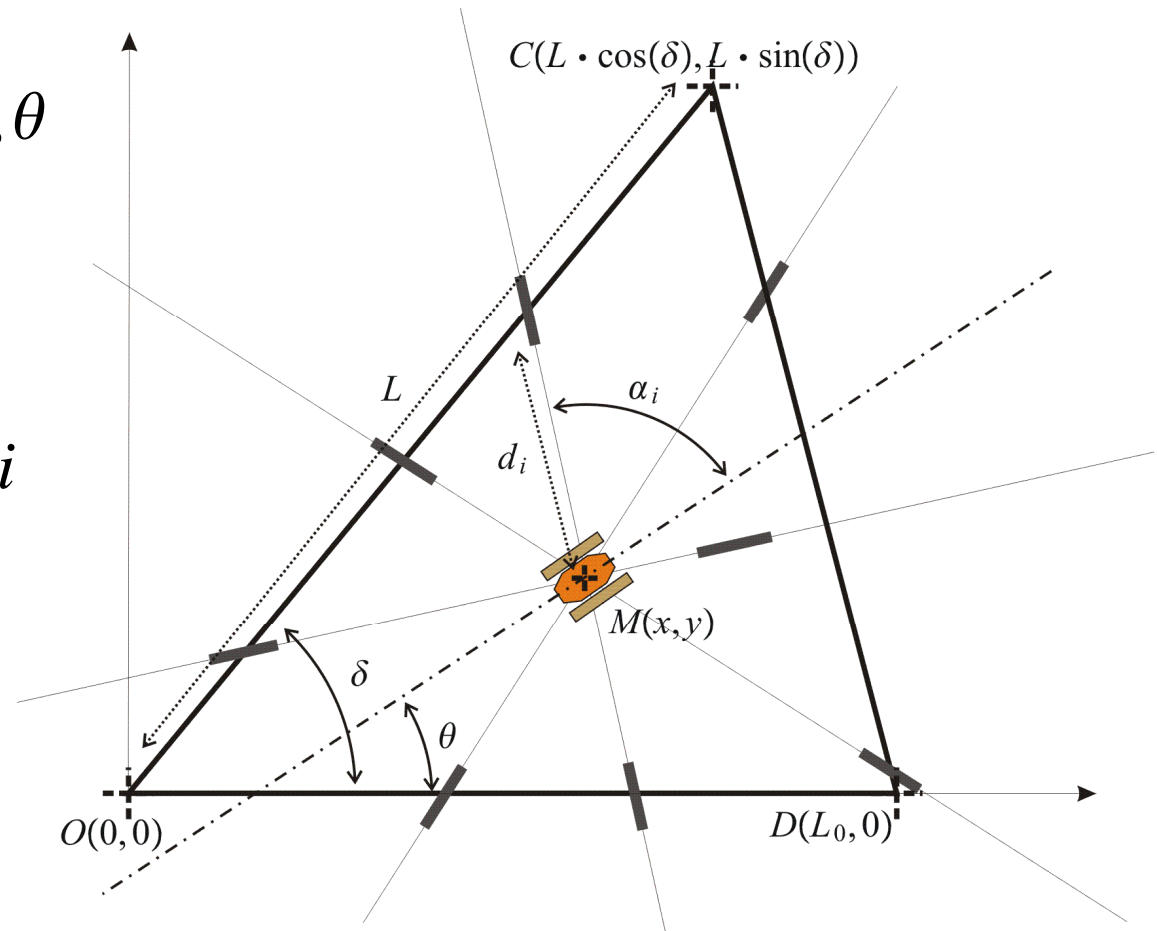
Simplified Problem

Unknown factors

- position x, y, θ
- map L, δ

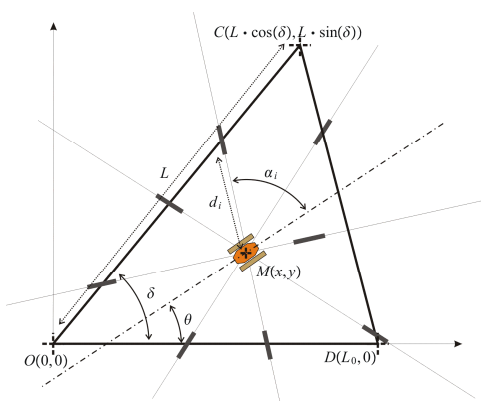
Data

- n telemetric measures d_i



Setting in equations

- We would like to set the SLAM problem in the form of a CSP (Constraint Satisfaction Problem) \rightarrow Solve the CSP using constraint propagation methods (Interval analysis)



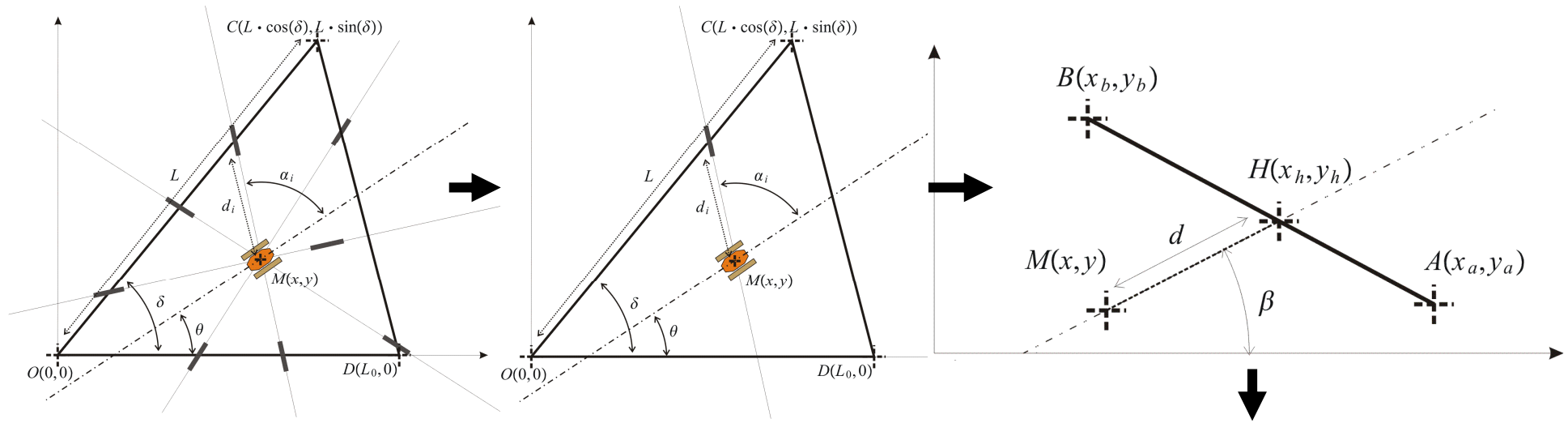
$$\left\{ \begin{array}{l} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \rightarrow \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right. \rightarrow$$

Interval analysis

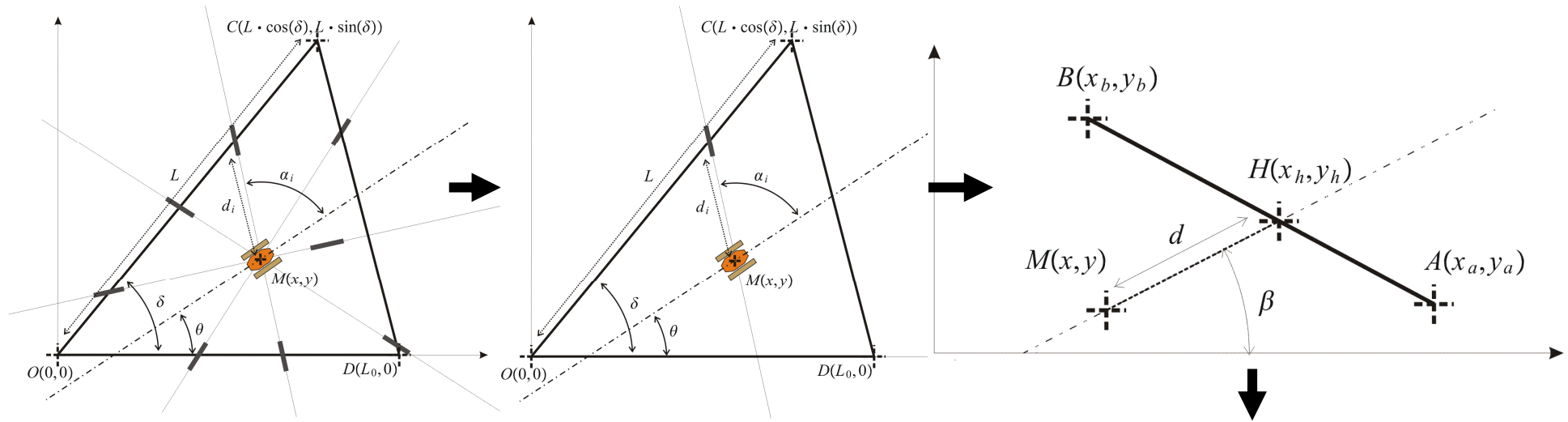
- Good **non-linear** equation handling
- **robust** with regards to outliers

Software Demonstration
in the end of the
presentation

Setting in equations : calculus details

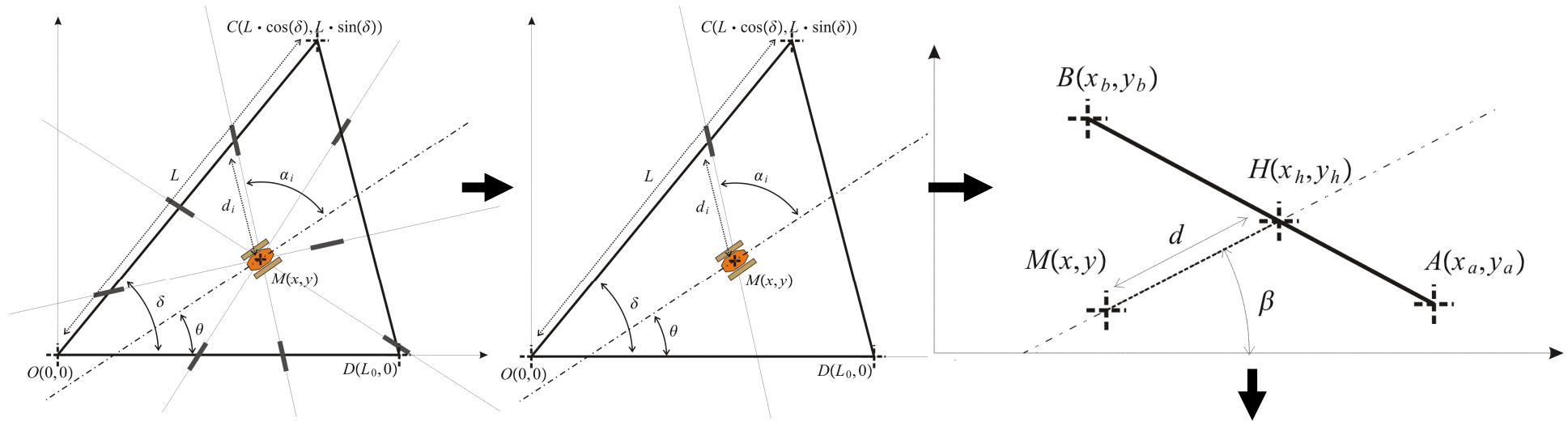


Setting in equations : calculus details



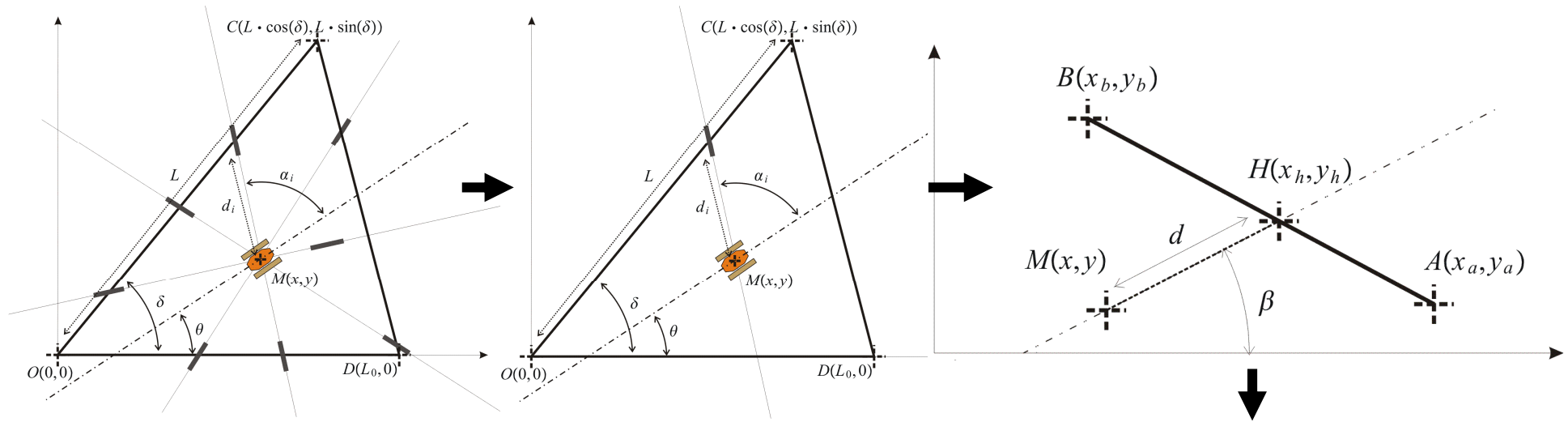
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Setting in equations : calculus details



$$\left\{ \begin{array}{l} \mathbf{f}_{i^{th_mesure}}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{i^{th_mesure}} : \mathbb{R}^q \rightarrow \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right. \quad \text{OR} \quad \left\{ \begin{array}{l} \mathbf{f}_{segment_ab}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{segment_ab} : \mathbb{R}^q \rightarrow \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right.$$

Setting in equations : calculus details



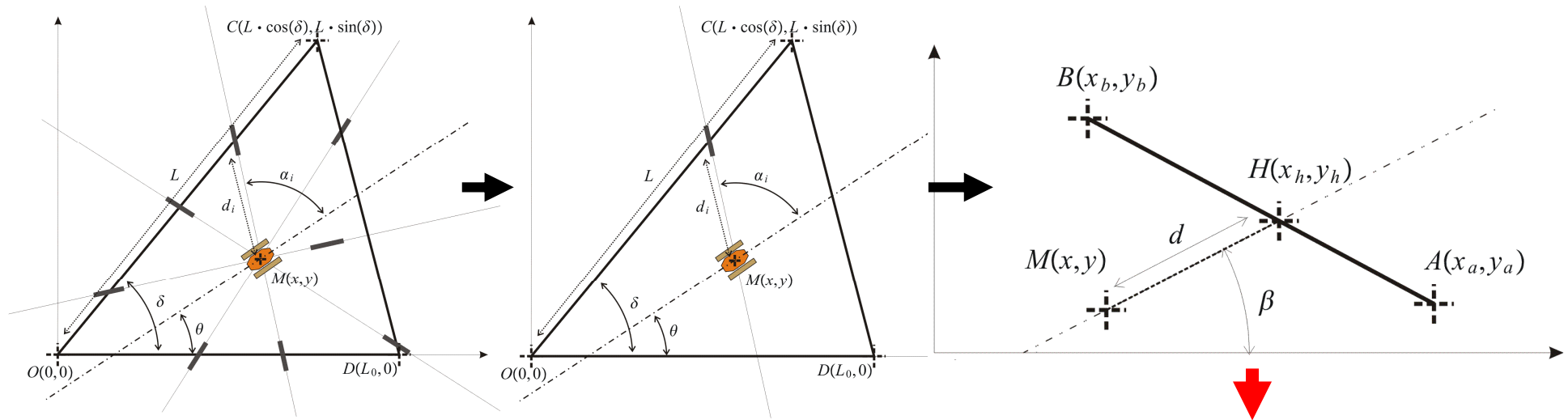
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$$\left\{ \begin{array}{l} \mathbf{f}_{i^{th_mesure}}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{i^{th_mesure}} : \mathbb{R}^q \rightarrow \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right.$$

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Setting in equations : calculus details



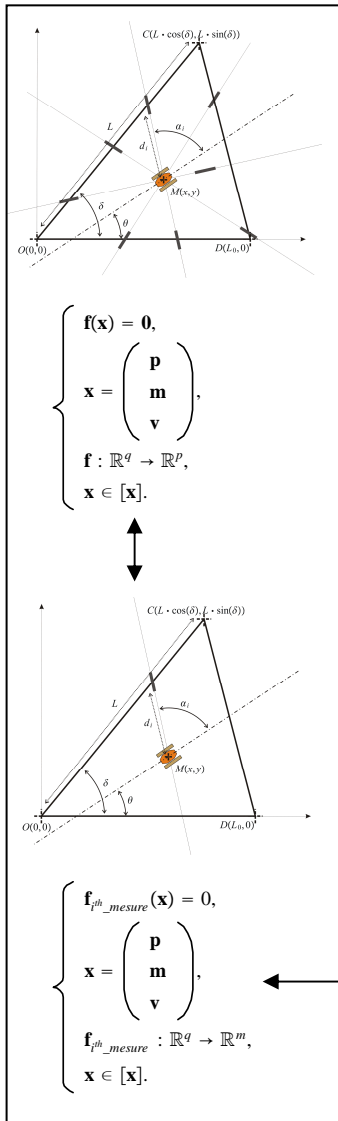
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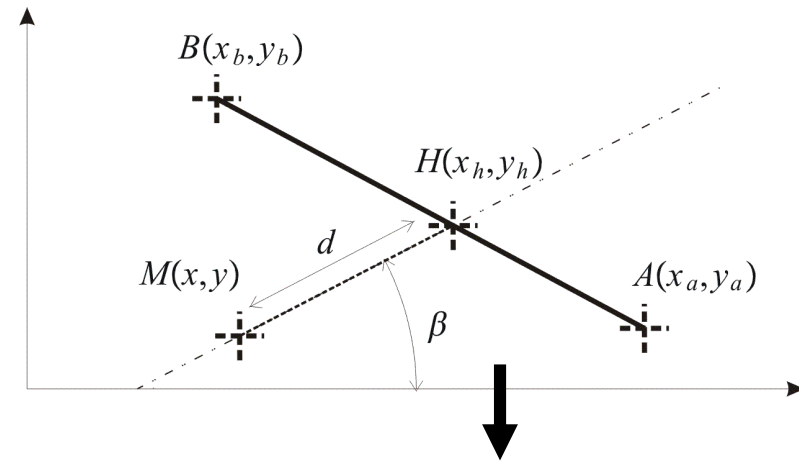
OR

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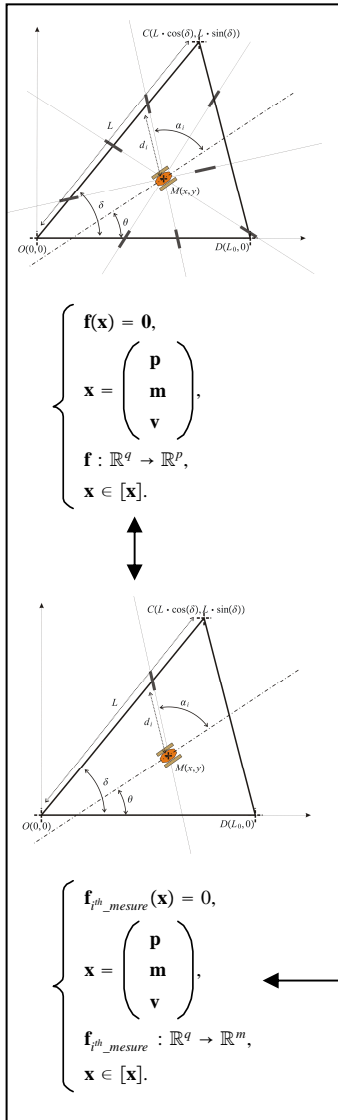
Setting in equations for each segment



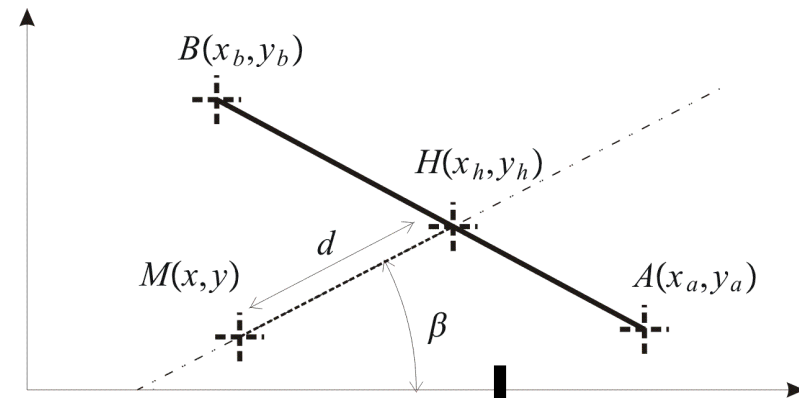
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Setting in equations for each segment



$$\left\{ \begin{array}{l} \mathbf{f}_{segment_ab}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{segment_ab}: \mathbb{R}^q \rightarrow \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right.$$



$$\det(\overrightarrow{AB}, \overrightarrow{AH}) = 0$$

$$\langle \overrightarrow{AH}, \overrightarrow{HB} \rangle > 0$$

$$\det(\overrightarrow{AB}, \overrightarrow{AM}) > 0$$

$$f_1(x, y, x_a, y_a, x_b, y_b, d, \beta) = 0$$

$$f_2(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p2} = 0$$

$$f_3(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p3} = 0$$

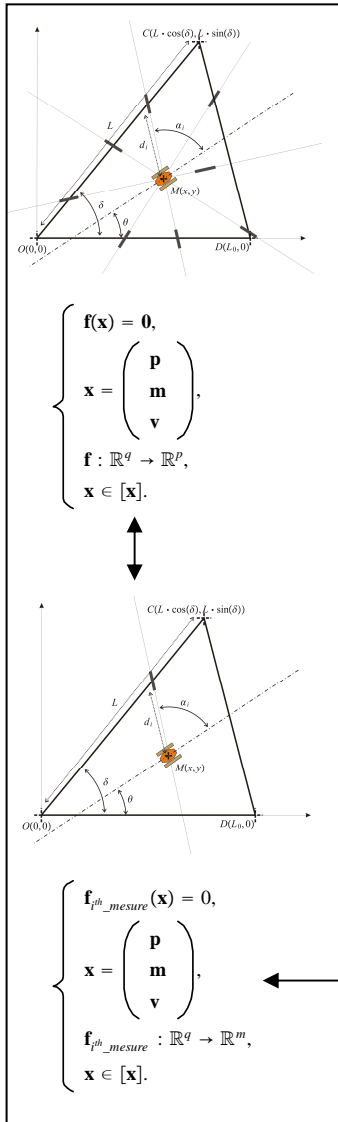
$$z_{p2} \in [0, +\infty], z_{p3} \in [0, +\infty]$$

$$f_1(x, y, x_a, y_a, x_b, y_b, d, \beta) = 0$$

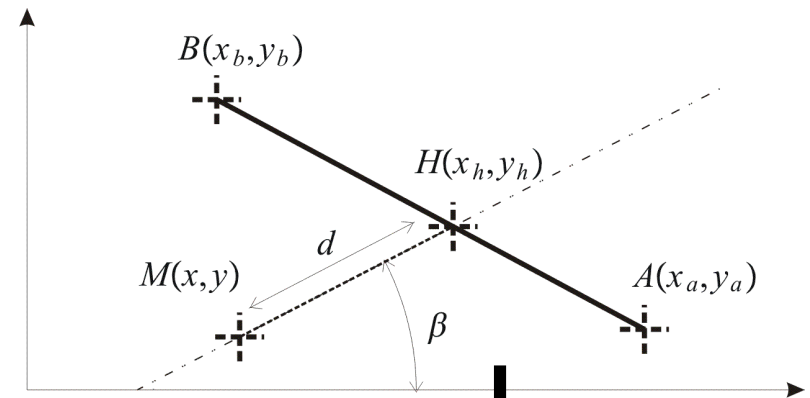
$$f_2(x, y, x_a, y_a, x_b, y_b, d, \beta) > 0$$

$$f_3(x, y, x_a, y_a, x_b, y_b, d, \beta) > 0$$

Setting in equations for each segment



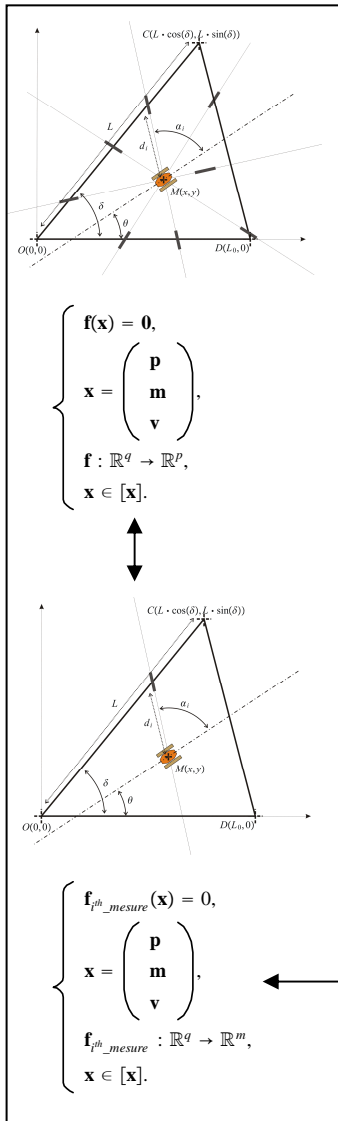
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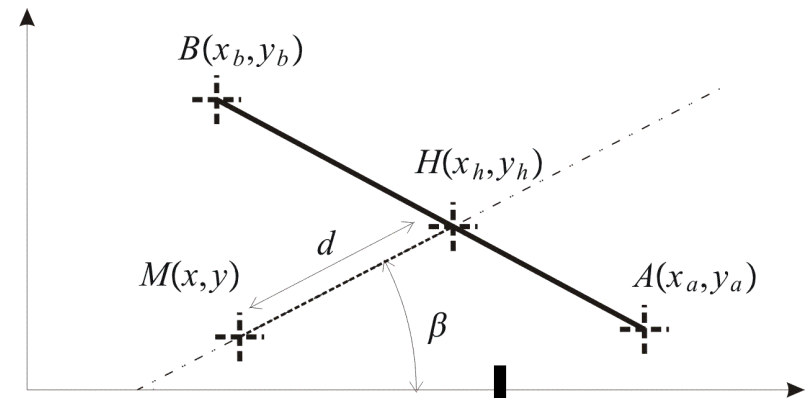
$$\begin{aligned} \det(\overrightarrow{AB}, \overrightarrow{AH}) &= 0 \\ \langle \overrightarrow{AH}, \overrightarrow{HB} \rangle &> 0 \\ \det(\overrightarrow{AB}, \overrightarrow{AM}) &> 0 \end{aligned}$$

$$\begin{aligned} f_1(x, y, x_a, y_a, x_b, y_b, d, \beta) &= 0 \\ \leftarrow f_2(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p2} &= 0 \\ f_3(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p3} &= 0 \\ z_{p2} \in [0, +\infty], z_{p3} \in [0, +\infty] & \end{aligned}$$

Setting in equations for each segment



$$\left\{ \begin{array}{l} \mathbf{f}_{segment_ab}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{segment_ab}: \mathbb{R}^q \rightarrow \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right.$$



$$\det(\vec{AB}, \vec{AH}) = 0$$

AND $\langle \vec{AH}, \vec{HB} \rangle > 0$

AND $\det(\vec{AB}, \vec{AM}) > 0$

$$f_1(x, y, x_a, y_a, x_b, y_b, d, \beta) = 0$$

← $f_2(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p2} = 0$

$$f_3(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p3} = 0$$

$$z_{p2} \in [0, +\infty], z_{p3} \in [0, +\infty]$$

Aparté on the AND

- The AND rule

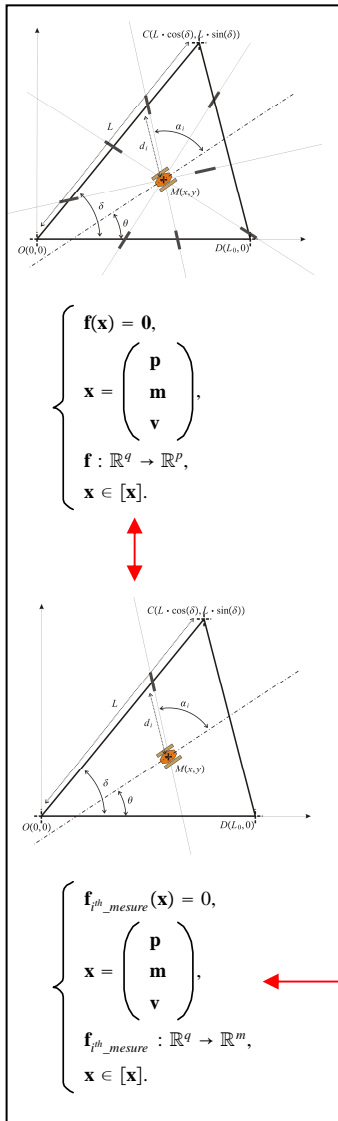
As an example if we want

$$X_1 = 0 \text{ **AND** } X_2 = 0 \text{ **AND** } X_3 = 0$$

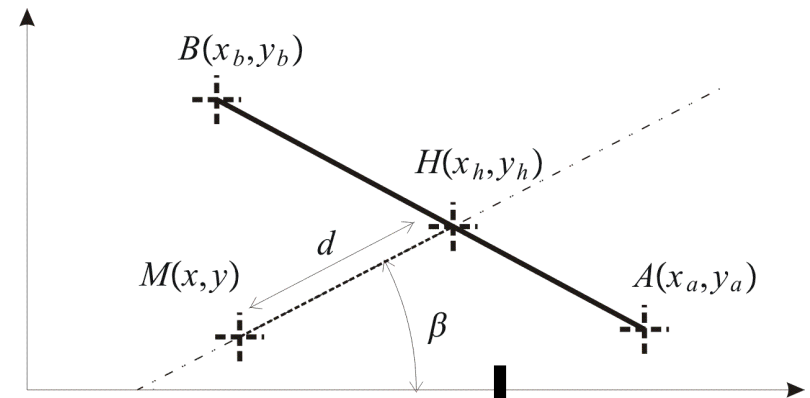
It is equivalent to say that

$$|X_1| + |X_2| + |X_3| = 0$$

Setting in equations for each segment



$$\left\{ \begin{array}{l} f_{segment_ab}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ f_{segment_ab}: \mathbb{R}^q \rightarrow \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right.$$



$$\det(\overrightarrow{AB}, \overrightarrow{AH}) = 0$$

AND $\langle \overrightarrow{AH}, \overrightarrow{HB} \rangle > 0$

AND $\det(\overrightarrow{AB}, \overrightarrow{AM}) > 0$

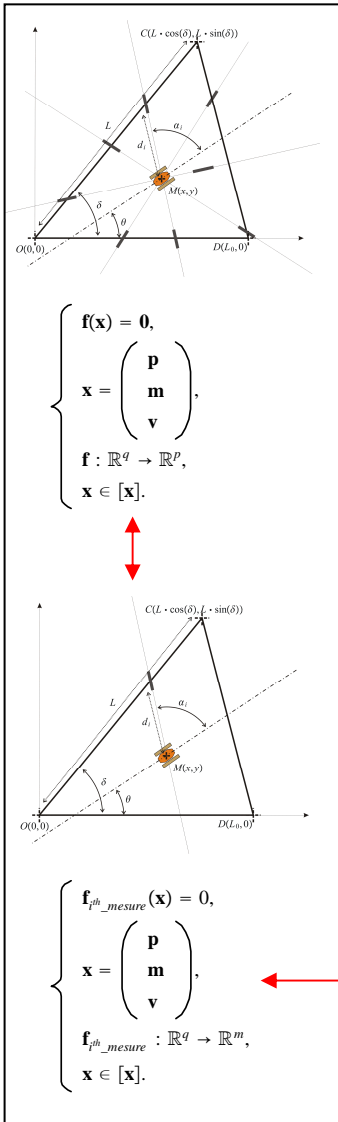
$$f_1(x, y, x_a, y_a, x_b, y_b, d, \beta) = 0$$

$$\leftarrow f_2(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p2} = 0$$

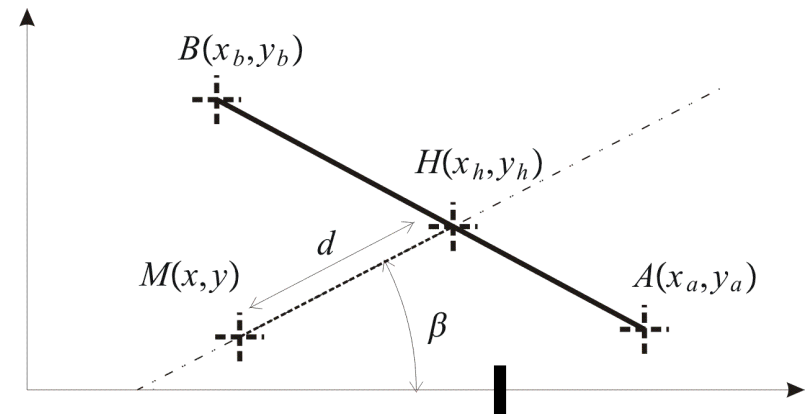
$$f_3(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p3} = 0$$

$$z_{p2} \in [0, +\infty], z_{p3} \in [0, +\infty]$$

Setting in equations for each segment



$$\left\{ \begin{array}{l} \mathbf{f}_{\text{segment}_{ab}}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{\text{segment}_{ab}}: \mathbb{R}^q \rightarrow \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right.$$



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$$f_1(x, y, x_a, y_a, x_b, y_b, d, \beta) = 0$$

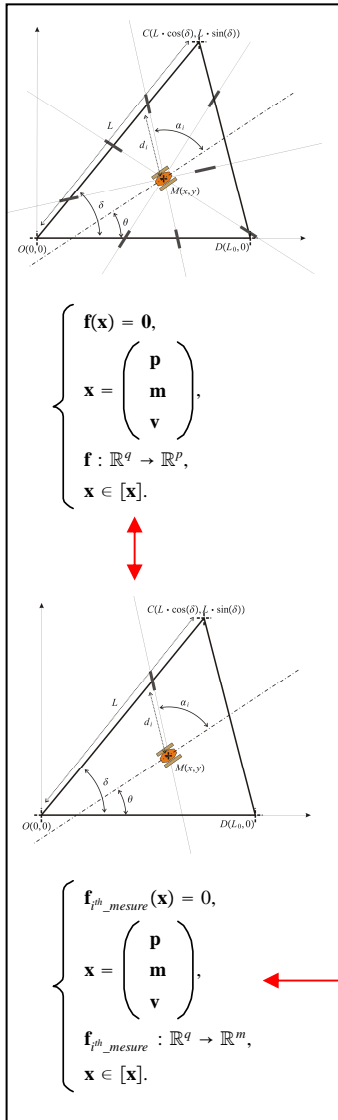
$$f_2(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p2} = 0$$

$$f_3(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p3} = 0$$

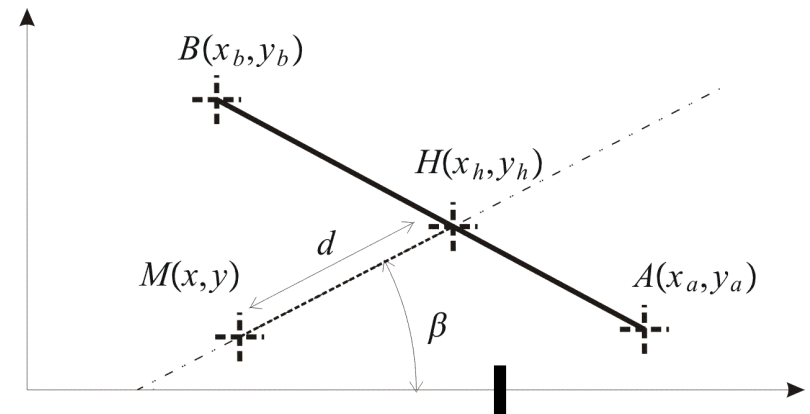
$$z_{p2} \in [0, +\infty], z_{p3} \in [0, +\infty]$$

$$\left\{ \begin{array}{l} f_1(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_1 = 0 \\ f_2(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p2} - z_2 = 0 \\ f_3(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p3} - z_3 = 0 \\ |z_1| + |z_2| + |z_3| - z_{ab} = 0 \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty] \\ z_1 \in [-\infty, +\infty], z_2 \in [-\infty, +\infty], z_3 \in [-\infty, +\infty] \\ z_{p2} \in [0, +\infty], z_{p3} \in [0, +\infty] \end{array} \right.$$

Setting in equations for each segment



$$\left\{ \begin{array}{l} \mathbf{f}_{segment_ab}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{segment_ab}: \mathbb{R}^q \rightarrow \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right.$$



$$\det(\overrightarrow{AB}, \overrightarrow{AH}) = 0$$

AND $\langle \overrightarrow{AH}, \overrightarrow{HB} \rangle > 0$

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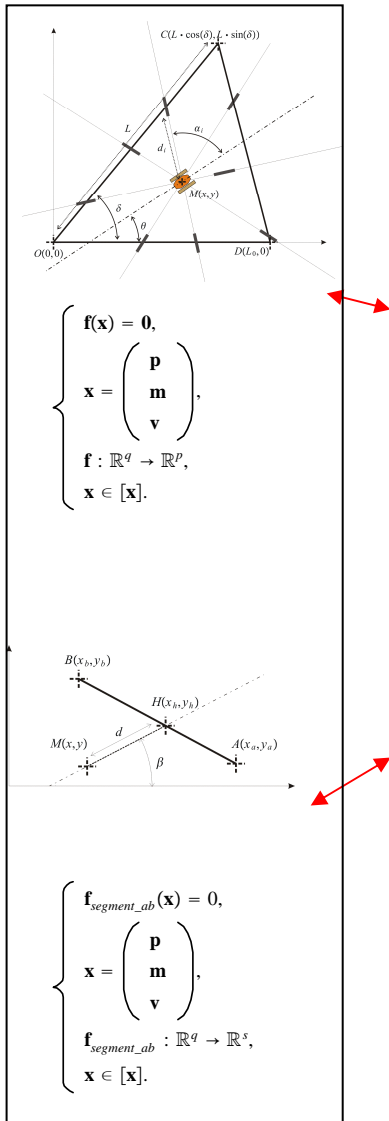
$$f_1(x, y, x_a, y_a, x_b, y_b, d, \beta) = 0$$

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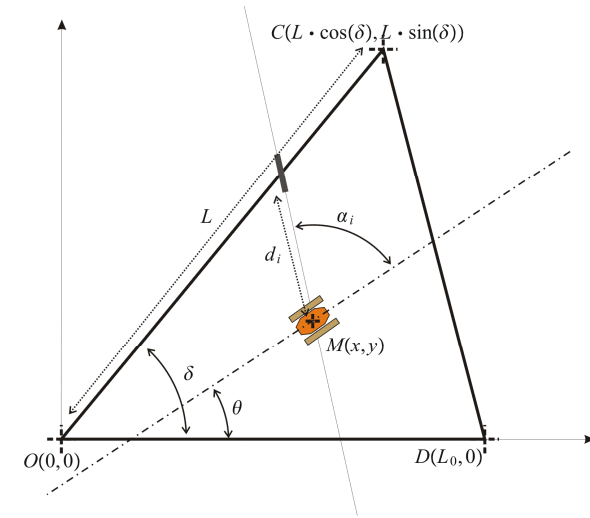
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$$z_{p2} \in [0, +\infty], z_{p3} \in [0, +\infty]$$

Setting in equations for each measure



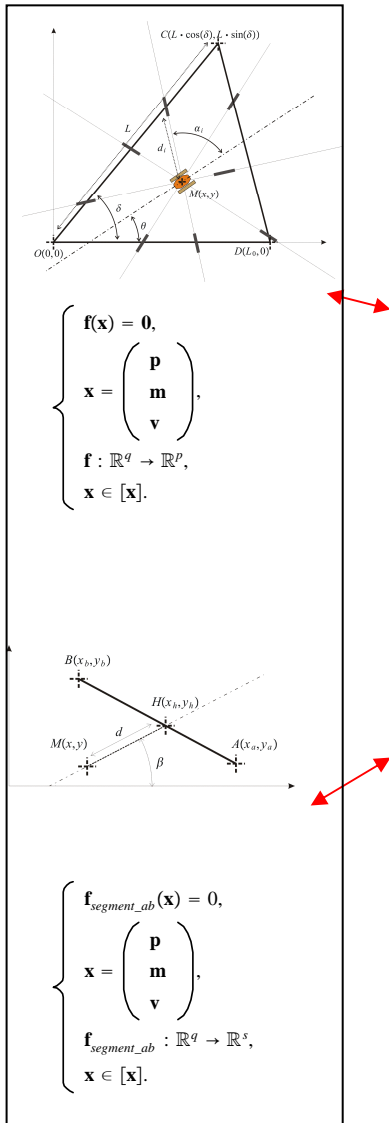
$$\left\{ \begin{array}{l} \mathbf{f}_{i^{th_measure}}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{i^{th_measure}} : \mathbb{R}^q \rightarrow \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right.$$



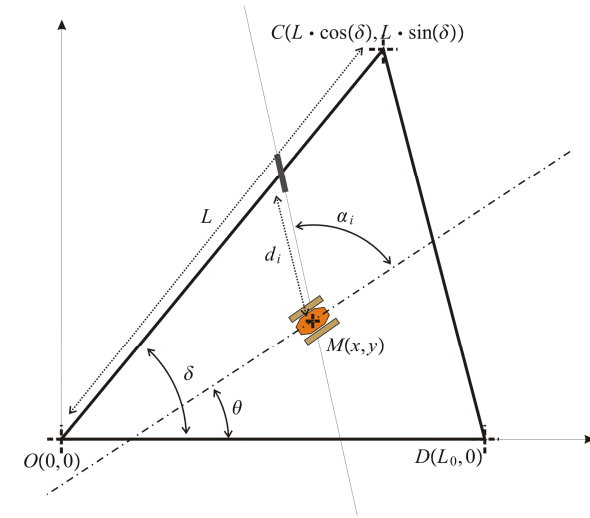
For each segment

$$\left\{ \begin{array}{l} \mathbf{f}_{seg_ab}(\mathbf{x}_{seg}, z_{ab}) = \mathbf{0} \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty], \\ \mathbf{x}_{seg} \in [\mathbf{x}_{seg}] \end{array} \right.$$

Setting in equations for each measure



$$\left\{ \begin{array}{l} \mathbf{f}_{i^{th_measure}}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{i^{th_measure}} : \mathbb{R}^q \rightarrow \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right.$$



For each segment

OR



$$\left\{ \begin{array}{l} \mathbf{f}_{seg_ab}(\mathbf{x}_{seg}, z_{ab}) = \mathbf{0} \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty], \\ \mathbf{x}_{seg} \in [\mathbf{x}_{seg}] \end{array} \right.$$

Aparté on the OR

- The OR rule

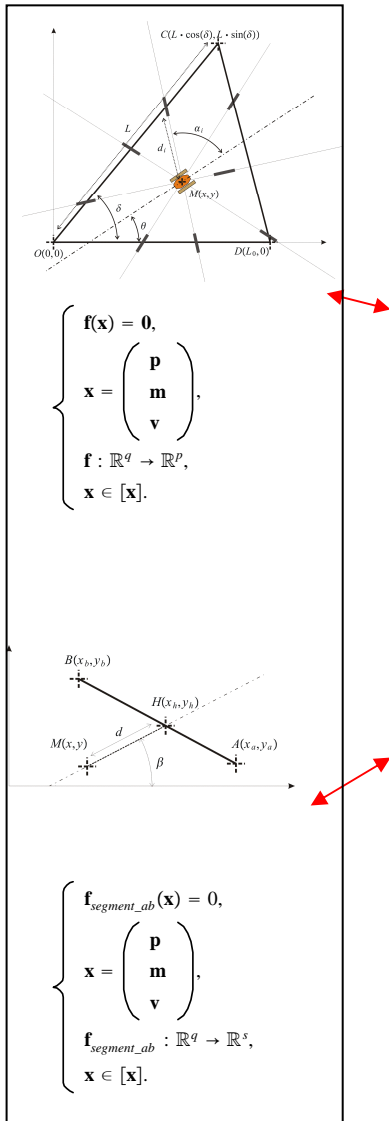
As an example if we want

$$X_1 = 0 \text{ OR } X_2 = 0 \text{ OR } X_3 = 0$$

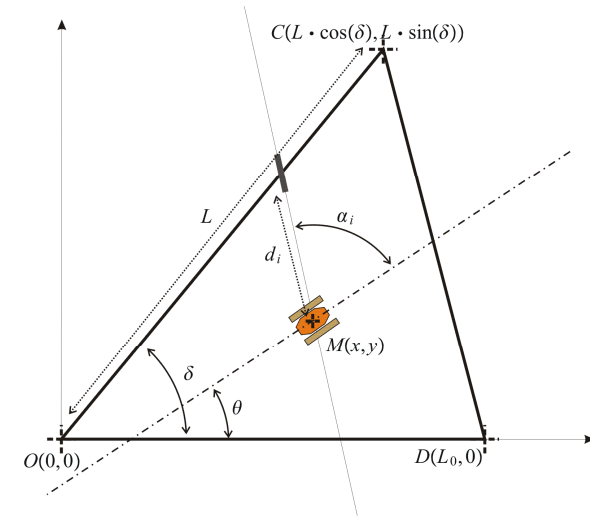
It is equivalent to say that

$$X_1 \cdot X_2 \cdot X_3 = 0$$

Setting in equations for each measure



$$\left\{ \begin{array}{l} \mathbf{f}_{i^{th_measure}}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{i^{th_measure}} : \mathbb{R}^q \rightarrow \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right.$$

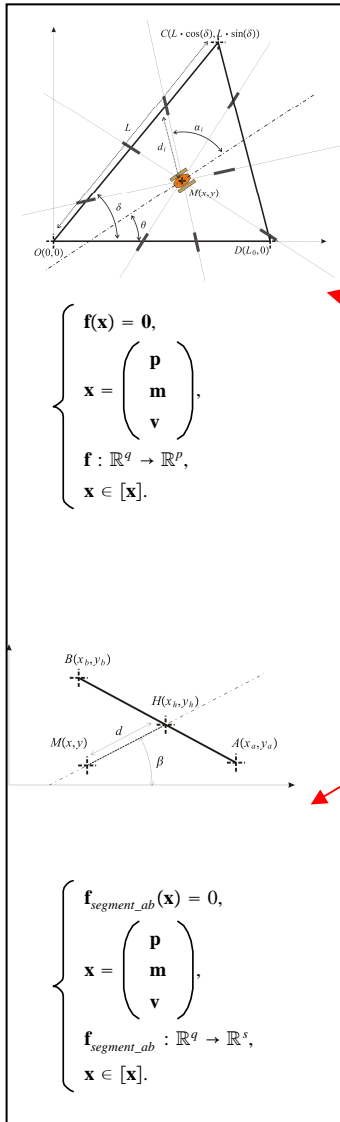


For each segment

OR

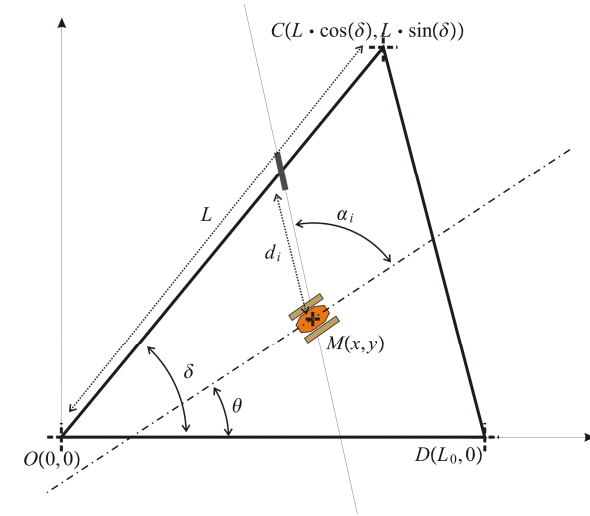
$$\left\{ \begin{array}{l} \mathbf{f}_{seg_ab}(\mathbf{x}_{seg}, z_{ab}) = \mathbf{0} \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty], \\ \mathbf{x}_{seg} \in [\mathbf{x}_{seg}] \end{array} \right.$$

Setting in equations for each measure



$$\left\{ \begin{array}{l} \mathbf{f}_{i^{th_measure}}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{i^{th_measure}} : \mathbb{R}^q \rightarrow \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right.$$

$$\left\{ \begin{array}{l} \mathbf{f}_{iseg_od}(\mathbf{x}, z_{iod}) = \mathbf{0} \\ \mathbf{f}_{iseg_dc}(\mathbf{x}, z_{idc}) = \mathbf{0} \\ \mathbf{f}_{iseg_co}(\mathbf{x}, z_{ico}) = \mathbf{0} \\ z_{iod} \cdot z_{idc} \cdot z_{ico} - z_i = 0 \\ z_i = 0? \\ z_i \in [-\infty, +\infty] \\ z_{iod} \in [-\infty, +\infty], z_{idc} \in [-\infty, +\infty], z_{ico} \in [-\infty, +\infty] \end{array} \right.$$



For each segment

OR



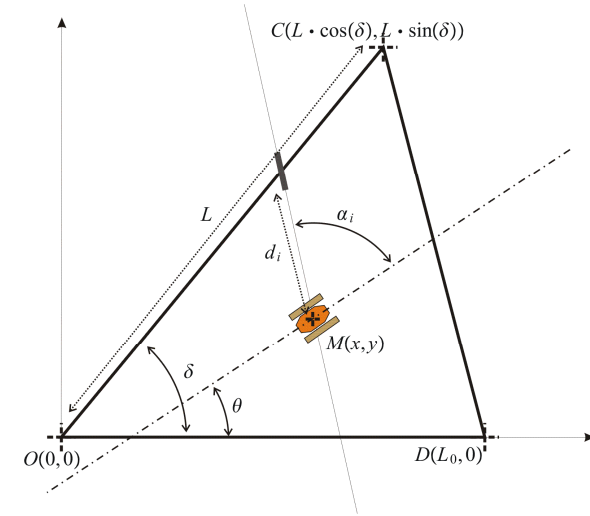
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Setting in equations for each measure

$\mathbf{f}(\mathbf{x}) = \mathbf{0},$
 $\mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix},$
 $\mathbf{f} : \mathbb{R}^q \rightarrow \mathbb{R}^p,$
 $\mathbf{x} \in [\mathbf{x}].$

$\mathbf{f}_{segment_ab}(\mathbf{x}) = 0,$
 $\mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix},$
 $\mathbf{f}_{segment_ab} : \mathbb{R}^q \rightarrow \mathbb{R}^s,$
 $\mathbf{x} \in [\mathbf{x}].$

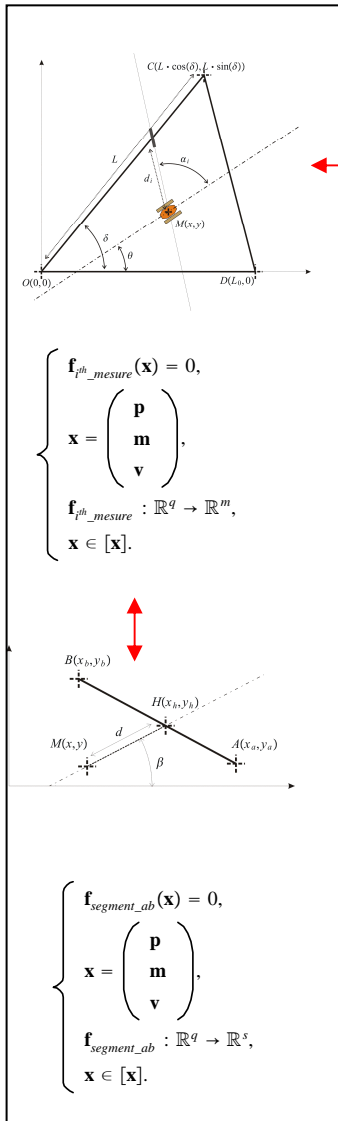
$$\left\{ \begin{array}{l} \mathbf{f}_{i^{th_measure}}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{i^{th_measure}} : \mathbb{R}^q \rightarrow \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right.$$



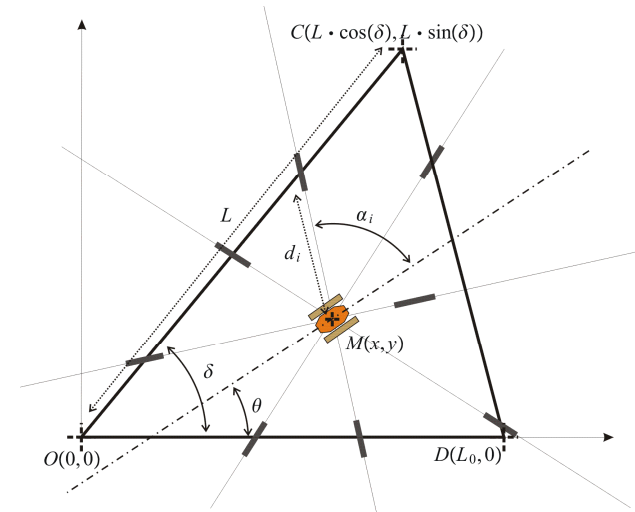
For each segment

$$\left\{ \begin{array}{l} \mathbf{f}_{seg_ab}(\mathbf{x}, z_{ab}) = \mathbf{0} \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty], \\ \mathbf{x} \in [\mathbf{x}] \end{array} \right.$$

Setting in the final equations



$$\left\{ \begin{array}{l} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \rightarrow \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right.$$



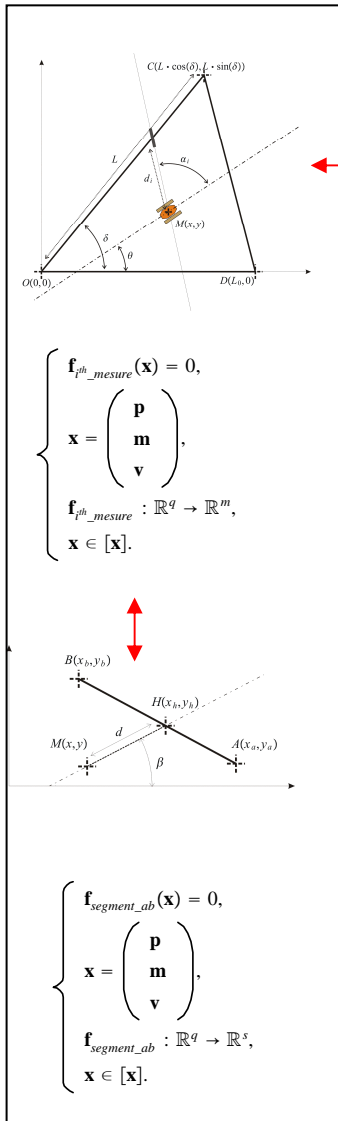
For each measure

Without outliers



$$\left\{ \begin{array}{l} \mathbf{f}_i(\mathbf{x}, z_i) = \mathbf{0} \\ z_i = 0? \\ z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{array} \right.$$

Setting in the final equations



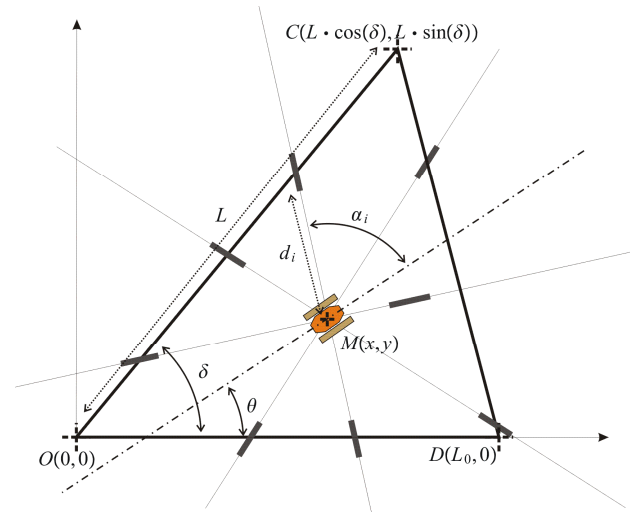
$$\left\{ \begin{array}{l} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \rightarrow \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right.$$

$$\left\{ \begin{array}{l} \mathbf{f}_1(\mathbf{x}, z_1) = \mathbf{0} \\ \dots \\ \mathbf{f}_i(\mathbf{x}, z_i) = \mathbf{0} \\ \dots \\ \mathbf{f}_n(\mathbf{x}, z_n) = \mathbf{0} \\ |z_1| + |z_2| + \dots + |z_n| = 0 \\ \forall i \in \{1..n\}, z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{array} \right.$$

Without outliers

AND

$$\left\{ \begin{array}{l} \mathbf{f}_i(\mathbf{x}, z_i) = \mathbf{0} \\ z_i = 0? \\ z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{array} \right.$$



Aparté on symmetric polynomials

- Basic symmetric polynomials

Definition $\phi_k(X_1, \dots, X_n) = \sum_{1 \leq i_1 < \dots < i_k \leq n} X_{i_1} X_{i_2} \dots X_{i_k}$

Example $\phi_0(X_1, \dots, X_4) = X_1 + X_2 + X_3 + X_4$ (AND)

$$\phi_1(X_1, \dots, X_4) = X_1X_2 + X_1X_3 + X_1X_4 + X_2X_3 + X_2X_4 + X_3X_4$$

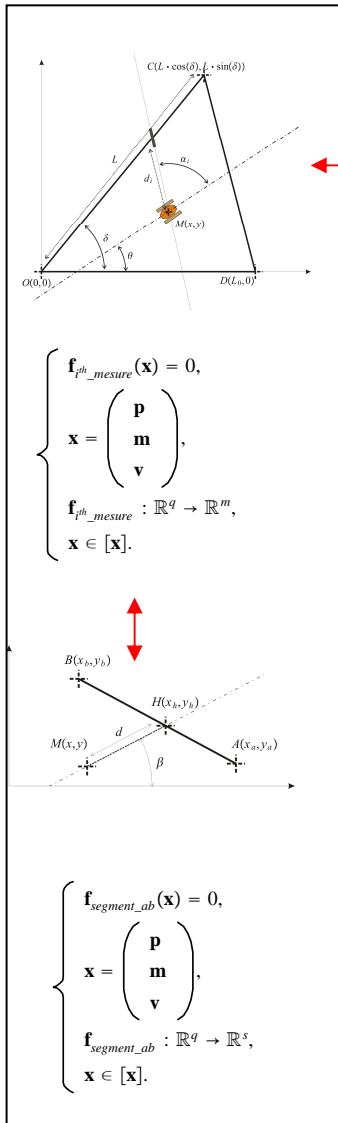
$$\phi_2(X_1, \dots, X_4) = X_1X_2X_3 + X_1X_2X_4 + X_1X_3X_4 + X_2X_3X_4$$

$$\phi_3(X_1, \dots, X_4) = X_1X_2X_3X_4$$
 (OR)

Interesting property $\forall i \in \{1, \dots, n\}, X_i \in [0, +\infty]$

If $\phi_\kappa(X_1, X_2, \dots, X_n) = 0$ then $n - \kappa$ variables among X_1, \dots, X_n are equal to zero

Setting in the final equations



$$\left\{ \begin{array}{l} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \rightarrow \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right.$$

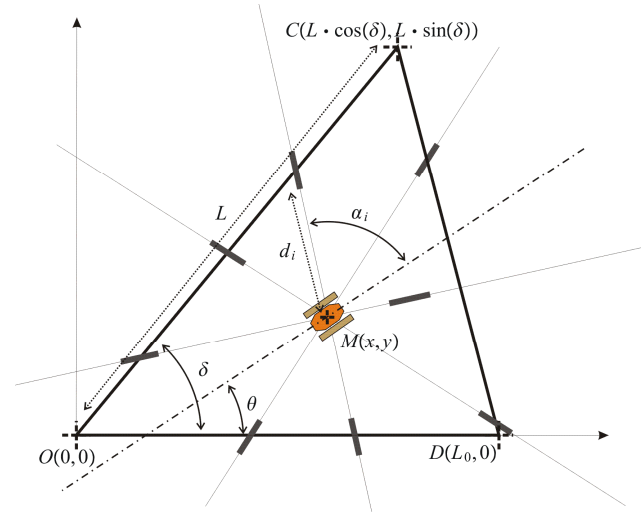
$$\left\{ \begin{array}{l} \mathbf{f}_1(\mathbf{x}, z_1) = 0 \\ \dots \\ \mathbf{f}_i(\mathbf{x}, z_n) = 0 \\ \dots \\ \mathbf{f}_n(\mathbf{x}, z_n) = 0 \end{array} \right.$$

$$\phi_K(|\mathbf{z}_1|, |\mathbf{z}_2|, \dots, |\mathbf{z}_n|) = \sum_{1 \leq i_0 < \dots < i_K \leq n} |\mathbf{z}_{i_1}| \cdot |\mathbf{z}_{i_2}| \cdot \dots \cdot |\mathbf{z}_{i_K}| = 0$$

$$\forall i \in \{1..n\}, z_i \in [-\infty, +\infty]$$

$$\mathbf{x} \in [\mathbf{x}]$$

k outlier



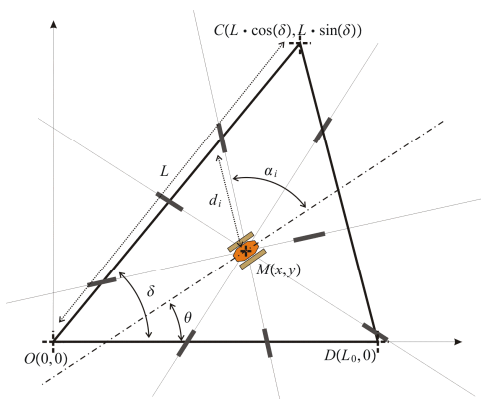
$$\left\{ \begin{array}{l} \mathbf{f}_i(\mathbf{x}, z_i) = 0 \\ z_i = 0? \\ z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{array} \right.$$

The final CSP

...
...
$L_0 \cdot (y + d_i \cdot \sin(\theta + \alpha_i)) - z_{iod1} = 0$
$(x + d_i \cdot \cos(\theta + \alpha_i))(L_0 - x - d_i \cdot \cos(\theta + \alpha_i)) - (y + d_i \cdot \sin(\theta + \alpha_i))^2 - z_{piod2} - z_{iod2} = 0$
$L_0 \cdot y - z_{piod3} - z_{iod3} = 0$
$ z_{iod1} + z_{iod2} + z_{iod3} - z_{iod} = 0$
$(L \sin(\delta) - L_0)(y + d_i \cdot \sin(\theta + \alpha_i)) + L \sin(\delta)(x + d_i \cdot \cos(\theta + \alpha_i) - L_0) - z_{idc1} = 0$
$(x + d_i \cdot \cos(\theta + \alpha_i) - L_0)(L \cos(\delta) - x - d_i \cdot \cos(\theta + \alpha_i)) + (y + d_i \cdot \sin(\theta + \alpha_i))(L \sin(\delta) - y - d_i \cdot \sin(\theta + \alpha_i)) - z_{pic2} - z_{idc2} = 0$
$(L \cos(\delta) - L_0) \cdot y - L \sin(\delta)(x - L_0) - z_{pic3} - z_{idc3} = 0$
$ z_{idc1} + z_{idc2} + z_{idc3} - z_{idc} = 0$
$-L \cos(\delta)(y + d_i \cdot \sin(\theta + \alpha_i) - L \sin(\delta)) - L \sin(\delta)(x + d_i \cdot \cos(\theta + \alpha_i) - L \cos(\delta)) - z_{ico1} = 0$
$(x + d_i \cdot \cos(\theta + \alpha_i) - L \cos(\delta))(-x - d_i \cdot \cos(\theta + \alpha_i)) + (y + d_i \cdot \sin(\theta + \alpha_i) - L \sin(\delta))(-y - d_i \cdot \sin(\theta + \alpha_i)) - z_{pico2} - z_{ico2} = 0$
$-L \cos(\delta)(y - L \sin(\delta)) + L \sin(\delta)(x - L \cos(\delta)) - z_{pico3} - z_{ico3} = 0$
$ z_{ico1} + z_{ico2} + z_{ico3} - z_{ico} = 0$
$z_{iod} \cdot z_{idc} \cdot z_{ico} - z_i = 0$
...
...
$\phi_{\kappa}(z_1 , z_2 , \dots, z_n) = \sum_{1 \leq i_0 < \dots < i_k \leq n} z_{i_1} \cdot z_{i_2} \cdot \dots \cdot z_{i_k} = 0$
$x \in [x], y \in [y], \theta \in [\theta], L \in [L], \delta \in [\delta]$
$\forall i \in \{1..n\}, d_i \in [d_i], \{z_{i*} \dots\} \subset [-\infty, +\infty], \{z_{pi*} \dots\} \subset [0, +\infty]$

Software demonstration

- We would like to set the SLAM problem in the form of a CSP (Constraint Satisfaction Problem) \rightarrow Solve the CSP using constraint propagation methods (Interval analysis)



$$\left\{ \begin{array}{l} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \rightarrow \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right. \rightarrow$$

Interval analysis

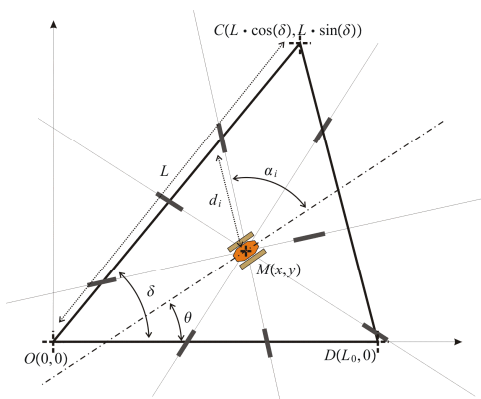
- Good non-linear equation handling
- **robust** with regards to outliers

Software Demonstration
in the end of the
presentation

??? Questions ???

Abstract

- We would like to set the SLAM problem in the form of a CSP (Constraint Satisfaction Problem) → Solve the CSP using constraint propagation methods (Interval analysis)



$$\left\{ \begin{array}{l} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \rightarrow \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{array} \right. \rightarrow$$

Interval analysis

- Good non-linear equation handling
- **robust** with regards to outliers

Software Demonstration
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