On the use of robust set inversion to characterize GNSS[1] protection levels

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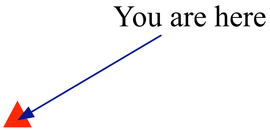
[1]Global Navigation Satellite Systems

Outline

1. Introduction
   - Motivation
   - GPS localization with code measurements
2. Measurements modeling and risk computation
   - Bounded-error measurement model
   - Integrity risk computation
3. Location zone computation
   - Protection zone
   - Robust set inversion
   - Fusion with altitude information

Set-membership GPS localization ?

- GPS is widely used for positionning services
- Confidence indicators are needed



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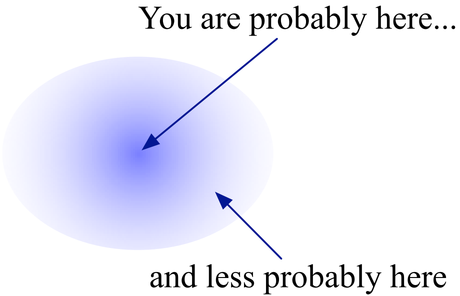

Set-membership GPS localization ?

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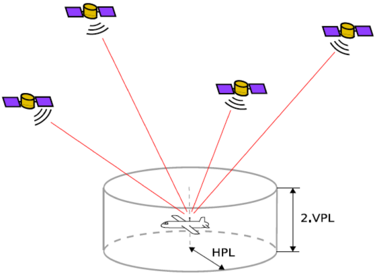
  - Protection levels (aviation standard): bounds on vertical (VPL) or horizontal (HPL) position error at a given integrity risk



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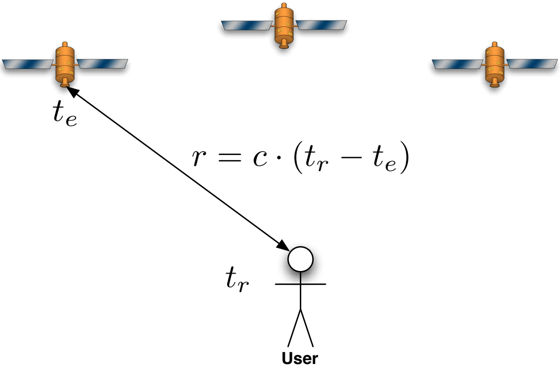
- The set-membership approach seems well suited for XPL computation:
  - Bounded error approach
  - Guaranteed algorithms with interval analysis: no risk added by the solver

Global Navigation Satellite Systems

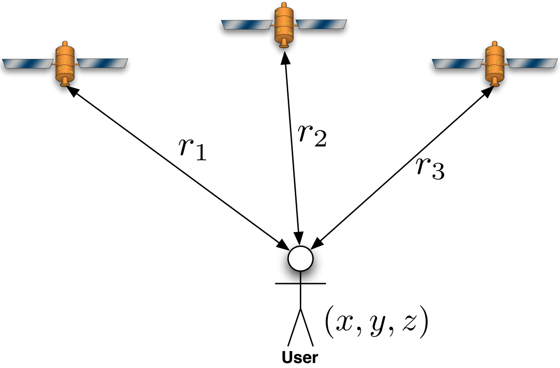
- Satellite constellations
  - NAVSTAR GPS: fully operational
  - GLONASS: being restored to full operation
  - Galileo: test bed (2 satellites)
  - COMPASS: test bed (1 satellite)
- 3 observables on L1 frequency band
  - code: pseudo random sequence ($\neq$ for each satellite)
  - phase
  - Doppler
- –> We will focus on code measurements

GPS localization



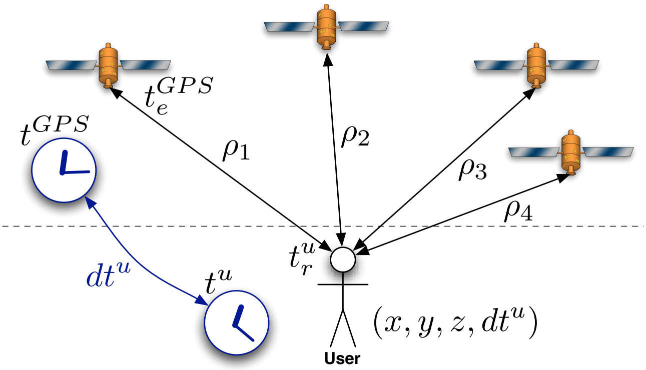
Time of flight measurement

GPS localization



Satellites positions are known (ephemeris): Trilateration

GPS localization

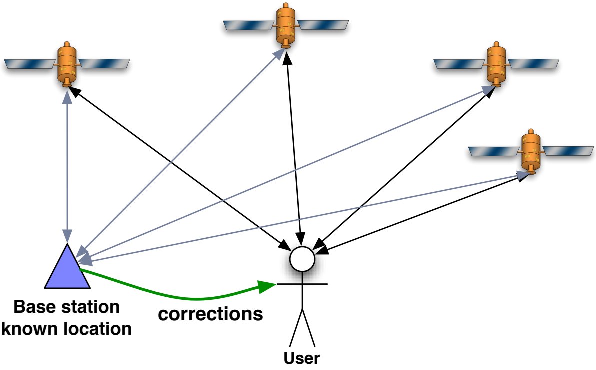


Unknown receiver clock time offset dt^u:

- no ranges but pseudoranges
- 4 satellites are needed

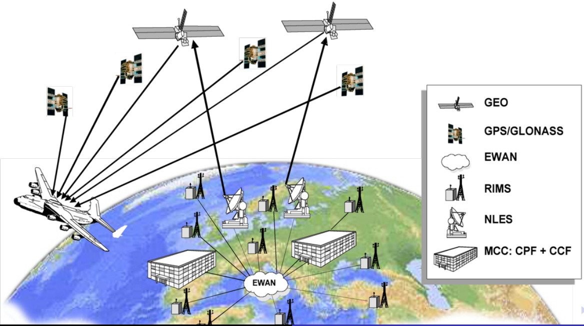
Enhancing accuracy: GBAS
Ground Based Augmentation Systems

- Local differential GPS with ground base



Enhancing accuracy: SBAS
Satellite Based Area Augmentation Systems

- Satellite Based Area Augmentation Systems: WAAS, EGNOS, MSAS
  - Pseudorange corrections
  - Integrity information
  - Characterisation of errors (Gaussian overbounding distribution)



Pseudorange observation equation

$$\begin{cases} \rho_1 &= \sqrt{(x - x_{s1})^2 + (y - y_{s1})^2 + (z - z_{s1})^2} + c \cdot dtu \\ \rho_2 &= \sqrt{(x - x_{s2})^2 + (y - y_{s2})^2 + (z - z_{s2})^2} + c \cdot dtu \\ & \ldots \\ \rho_p &= \sqrt{(x - x_{sp})^2 + (y - y_{sp})^2 + (z - z_{sp})^2} + c \cdot dtu \end{cases}$$

- Satellites broadcast IDs -> no data association needed
- $x_{s_i}, y_{s_i}, z_{s_i}$ are satellite positions (broadcast)
- ρ_i are corrected pseudoranges:
    - satellite clock bias
    - relativistic effects
    - ionosphere and troposphere propagation delays

Bounded-error measurement model

Positions of the satellites

- Ephemeris data are broadcast by satellites
- Satellites' positions computed from ephemeris data
- Orbits are not precisely known:
  - monitoring error
  - trajectory tracking error
  - quantization error

$$\begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix} \in \begin{pmatrix} x_s^{broadcast} \\ y_s^{broadcast} \\ z_s^{broadcast} \end{pmatrix} + \begin{pmatrix} [e_x^-; e_x^+] \\ [e_y^-; e_y^+] \\ [e_z^-; e_z^+] \end{pmatrix}$$

–> Satellite positions can be represented as boxes

Bounded-error measurement model
Pseudorange measurements

Corrected pseudoranges are high level data

- Pseudoranges corrections are inaccurate
- The receiver makes measurement errors

$$\rho_i \in \rho_i^{meas} + [e_{\rho_i}^-; e_{\rho_i}^+]$$

–> each measurement = an interval
–> measurement vector is a box $[\mathbf{y}_{\mathbf{meas}}]$

How to choose the bounds ?

- m measurements
- At most q measurements can be wrong ($e_{\rho_i} \notin [e^-_{\rho_i}; e^+_{\rho_i}]$)
- The solution $\mathbb{X}$ has to contain the true location $\mathbf{x}$ with a probability π
  - The integrity risk of $\mathbf{x}$ not being inside $\mathbb{X}$ is $r = 1 - \pi$
- We know the pdf of e_{ρ_i} (from EGNOS)

Risk computation

- Let p be the prior probability of an inlier:

$$p = P(e_{\rho_i} \in [e^-_{\rho_i}; e^+_{\rho_i}])$$

- Probability of exactly k inliers out of m measurements: Binomial law

$$P(n_{inliers} = k) = \binom{m}{k} p^k (1-p)^{m-k}$$

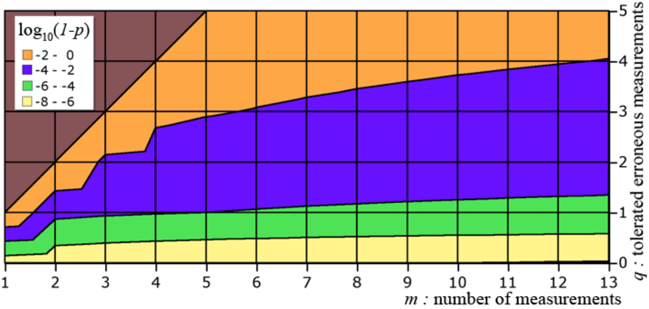
$$\text{where } \binom{m}{k} = \frac{m!}{k!(m-k)!}$$

- Probability of at most q outliers (at least $m - q$ inliers):

$$P(n_{inliers} \geq m - q) = \pi = 1 - r = \sum_{k=m-q}^{m} \binom{m}{k} p^k (1-p)^{m-k}$$

Specification of p to meet a risk requirement

Given m, q and r, we compute p such as
$1 - r = \sum_{k=m-q}^{m} \binom{m}{k} p^k (1-p)^{m-k}$.



$\log_{10}$ of $(1-p)$ to achieve an integrity risk r of 10^{-7}

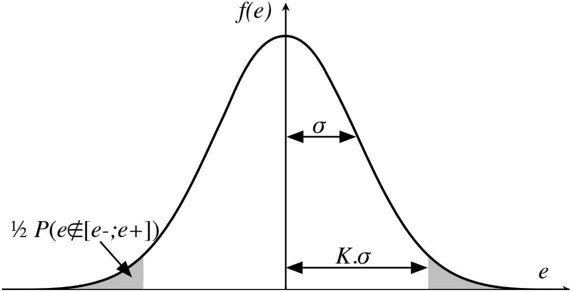
Setting measurement error bounds

The usage in GNSS is overbounding Gaussians: $e_{\rho_i} \sim \mathcal{N}(0, \sigma_{\rho_i})$

- σ_{ρ_i} from EGNOS

$$[e^-_{\rho_i}; e^+_{\rho_i}] = [K \cdot \sigma_y, -K \cdot \sigma_y] \quad \text{with } K = -\Phi^{-1}\left(\frac{1-p}{2}\right)$$

$$[\rho_i] = \rho_i^{meas} + [e^-_{\rho_i}; e^+_{\rho_i}]$$



Guaranteed solver

- Set Inversion Via Interval Analysis algorithm is used to compute location zones.
- We compute an outer approximation $\overline{\mathbb{X}}$ of the solution set $\mathbb{X}$.
- Solver is guaranteed to keep all solutions –> $\mathbb{X} \subseteq \overline{\mathbb{X}}$

$$n_{inliers} \geq m - q \Rightarrow x \in \mathbb{X} \Rightarrow x \in \overline{\mathbb{X}}$$

$$P(x \in \overline{\mathbb{X}}) \geq P(x \in \mathbb{X}) \geq \pi$$

$$P(x \notin \overline{\mathbb{X}}) \leq P(x \notin \mathbb{X}) \leq r$$

- Integrity risk of the computed protection zone is less than the risk r chosen when setting error bounds

Protection zone

Simple case: no outliers tolerated

The set $\mathbb{X}$ of all locations compatible with the measurements and the satellite positions intervals:

$$\mathbb{X} = \Big\{ (x, y, z, cdtu) \in \mathbb{R}^4, \forall i = 1 \dots m, \\ \exists \rho_i \in [\rho_i], \exists (x_i^{\mathrm{s}}, y_i^{\mathrm{s}}, z_i^{\mathrm{s}}) \in [\mathbf{x}_i^{\mathrm{s}}], \\ \rho_i = \sqrt{(x - x_i^{\mathrm{s}})^2 + (y - y_i^{\mathrm{s}})^2 + (x - y_i^{\mathrm{s}})^2} + cdtu \Big\}$$

Solved using SIVIA with interval constraint propagation

Protection zone

Simple case: no outliers tolerated. $r = 10^{-7}$

Demo

Robust set inversion

- Let $\mathbb{X}_i$ be the solution set generated by only pseudorange i.
- The q-relaxed intersection of m sets is the set of solutions compatible with at least $m - q$ pseudoranges.
- $\bigcap^{\{q\}} \mathbb{X}_i$ remains consistent for up to q outliers

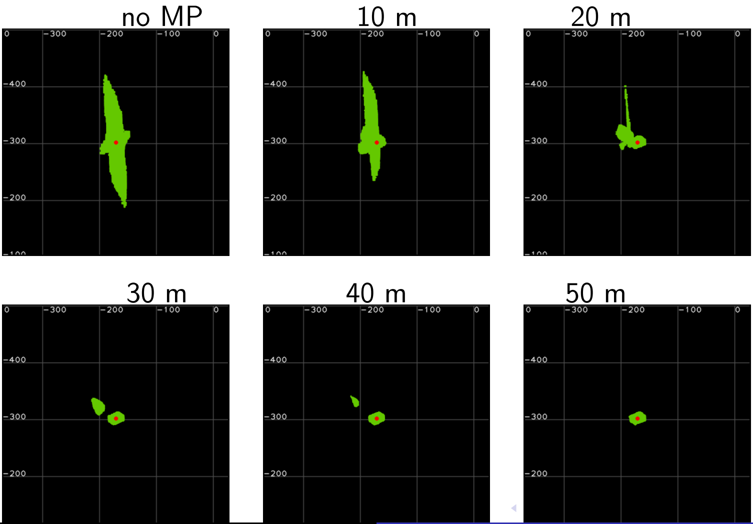
Protection zone

Simple case: no outliers tolerated. $r = 10^{-7}$

Demo

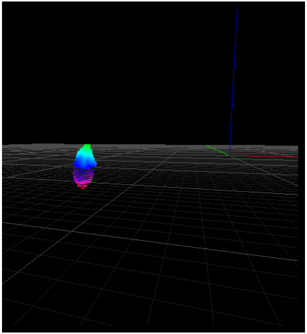
Robustness

(X,Y) projection. $q = 1$, $r = 5 \cdot 10^{-9}$. One measurement is affected by multipath

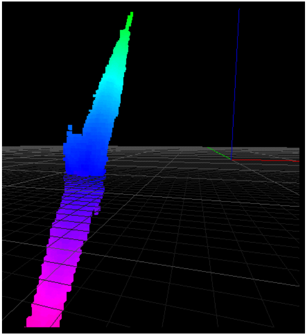


Protection zone comparison

Same measurements. All inliers. $r = 5 \cdot 10^{-9}$



Standard intersection



1-relaxed intersection

Adding redundancy

- Robust solver needs data redundancy
- In urban areas:
  - fewer GPS measurements (masking)
  - more erroneous measurements (multipath)
- We need other sources of data to ensure redundancy:
  - Other constellations (GALILEO, GLONASS)
  - Altitude (Digital Elevation Model)
  - Odometry
  - Map

Using altitude information

Altitude is known within ± 14m –> Constraint is directly applied to the prior box before calling solver.

Demo

Summary

- Interval methods allow computation of a location zone with a guaranteed integrity risk
- Integrity risk can be specified by proper adjustment of the measurement error bounds
- Robustness to erroneous measurements can easily be achieved, without loss of guarantee
- Fusion with other data enables tighter location zone computation
- Experimental results in real time with real GPS data. Parallelized solver for multicore support.

- Outlook
  - Data fusion with odometry and inertial sensors
  - Use of Doppler measurements
  - Map matching