An Expressive Mechanism for Auctions on the Web

Paul Dütting[1], Monika Henzinger[2], and Ingmar Weber[3]

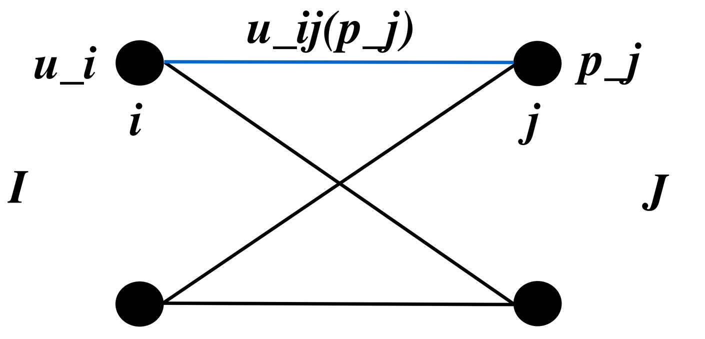
[1]Ecole Polytechnique Fédérale de Lausanne (EPFL), Switzerland

[2]University of Vienna, Austria

[3]Yahoo! Research Barcelona, Spain

paul.duetting@epfl.ch

Problem



Input:

- Set of bidders $\boldsymbol{I}$ with $|\boldsymbol{I}| = \boldsymbol{n}$ and set of items $\boldsymbol{J}$ with $|\boldsymbol{J}| = \boldsymbol{m}$
- Utility functions $\boldsymbol{u_{i,j}(p_j)}$ that are piece-wise linear and may have non-identical slopes and multiple discontinuities
- Outside options $\boldsymbol{o_i}$ and reserve prices $\boldsymbol{r_j}$

Output:

- Outcome $(\boldsymbol{\mu}, \boldsymbol{p})$ consisting of a matching $\boldsymbol{\mu}$ and prices $\boldsymbol{p}$
- Envy free iff $\boldsymbol{u_{i,j}(p_j) \geq u_{i,k}(p,k)}$ for all $(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{\mu}$ and all $\boldsymbol{k}$
- Bidder optimal iff $\boldsymbol{u_i \geq u'_i}$ for all $\boldsymbol{i}$ and all envy free outcomes $(\boldsymbol{\mu'}, \boldsymbol{p'})$

Motivation

Per Click vs. Per Impression Valuations

(This example motivates non-identical slopes.)

- Suppose that the mechanism uses linear utilities with identical slopes and that it computes per-click prices, i.e., $\boldsymbol{u_{i,j}(p_j^{click}) = v_{i,j} - p_j^{click}}$
- Suppose that per-click bidders report $\boldsymbol{v_{i,j} = v_{i,j}^{click}}$ and that per-impression bidders report $\boldsymbol{v_{i,j} = v_{i,j}^{imp}/ctr_{i,j}}$, where $\boldsymbol{ctr_{i,j}}$ is the click through rate
- The mechanism computes an outcome $(\boldsymbol{\mu}, \boldsymbol{p^{click}})$ that is envy free, i.e., $\boldsymbol{v_{i,j} - p_j^{click} \geq v_{i,k} - p_k^{click}}$ for all $(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{\mu}$ and all $\boldsymbol{k}$
- This is enough to ensure $\boldsymbol{v_{i,j}^{click} - p_j^{click} \geq v_{i,k}^{click} - p_k^{click}}$, but it implies $\boldsymbol{v_{i,j}^{imp} - p_j^{imp} \geq v_{i,k}^{imp} - p_k^{imp}}$ only if $\boldsymbol{ctr_{i,j}/ctr_{i,k} < 1}$

Soft and Hard Budgets

(This example motivates non-identical slopes and multiple discontinuities.)

- Suppose that bidder $\boldsymbol{i}$ has a limited amount of cash $\boldsymbol{c}$
- Suppose that he is willing to take out a loan with a maximum amount of $\boldsymbol{a}$, a fixed fee of $\boldsymbol{f}$, and an interest rate of $\boldsymbol{r}$
- Suppose that he is not willing to pay more than $\boldsymbol{b < c + a}$

Risk Aversion

(This example motivates non-linear utilities.)

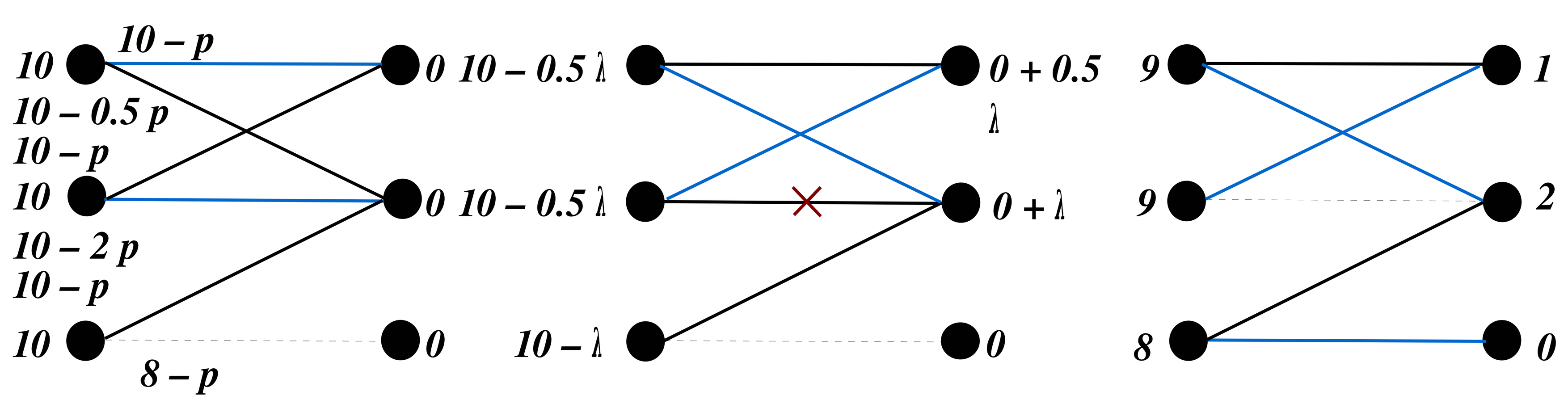
- A higher price might be associated with a higher risk of not having sufficient money to pay for unforeseen events such as repairs

Our Results

Mechanism for Piece-Wise Linear Utilities

- We give a polynomial-time mechanism for piece-wise linear utilities with non-identical slopes and multiple discontinuities

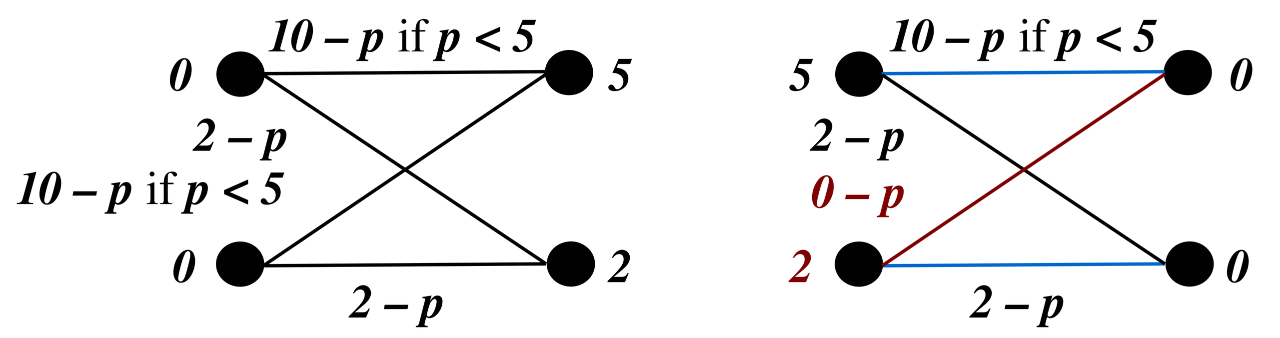
Example: Price Update



Characterization of Incentive Compatible Mechanisms

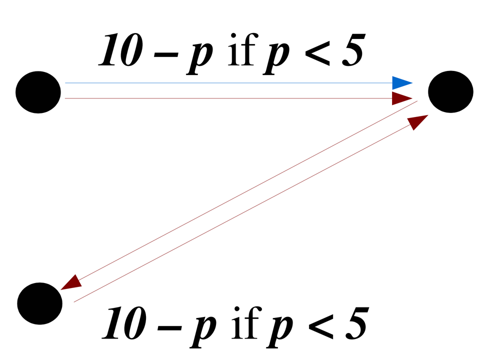
- No mechanism that computes a bidder optimal outcome $(\boldsymbol{\mu}, \boldsymbol{p})$ is incentive compatible, even if the utilities are linear, have identical slopes, and have only a single publicly known discontinuity [Aggarwal et al.]

Example: Lying Pays Off



- We show how to generalize the general position concept of [Aggarwal et al.] and prove that for inputs in general position any mechanism that computes a bidder optimal outcome $(\boldsymbol{\mu}, \boldsymbol{p})$ is incentive compatible

Example: General Position



Blue and red walk have the same weight ↯

- For inputs in general position the bidder optimal outcome $(\boldsymbol{\mu}, \boldsymbol{p})$ computed by our mechanism is a competitive equilibrium

General Utilities via Approximation

- We show how to compute a $\boldsymbol{\gamma}$-bidder optimal outcome $(\boldsymbol{\mu}, \boldsymbol{p})$ for non-linear, continuous utility functions via piece-wise linear approximation in time linear in $\boldsymbol{\sqrt{1/\gamma}}$

Hardness Results for Externalities

- We show that if the utility functions depend (a) on the matching $\boldsymbol{\mu}$ or (b) on the whole vector of prices $\boldsymbol{p}$, then it is $\boldsymbol{\mathcal{NP}}$-complete to decide whether there exists an envy free outcome $(\boldsymbol{\mu}, \boldsymbol{p})$ with sum of the utilities at least $\boldsymbol{K}$
- The first result is established through a reduction from **3SAT**, the second result is established through a reduction from **MAX-NASH**

Future Work

- It would be interesting to push the "expressiveness frontier" even further
- It would be nice to have a more mature "theory of expressiveness"

References:

G. Aggarwal, S. Muthukrishnan, D. Pál, M. Pál. General Auction Mechanism for Search Advertising, WWW, 241-250, 2009.

N. Chen, X. Deng, A. Ghosh. Competitive Equilibria in Matching Markets with Budgets, SIGecom Exchanges, 9.1, 1-5, 2010.

P. Dütting, M. Henzinger, I. Weber. An Expressive Mechanism for Auctions on the Web. Technical Report, 2010.