How much is your personal recommendation worth?

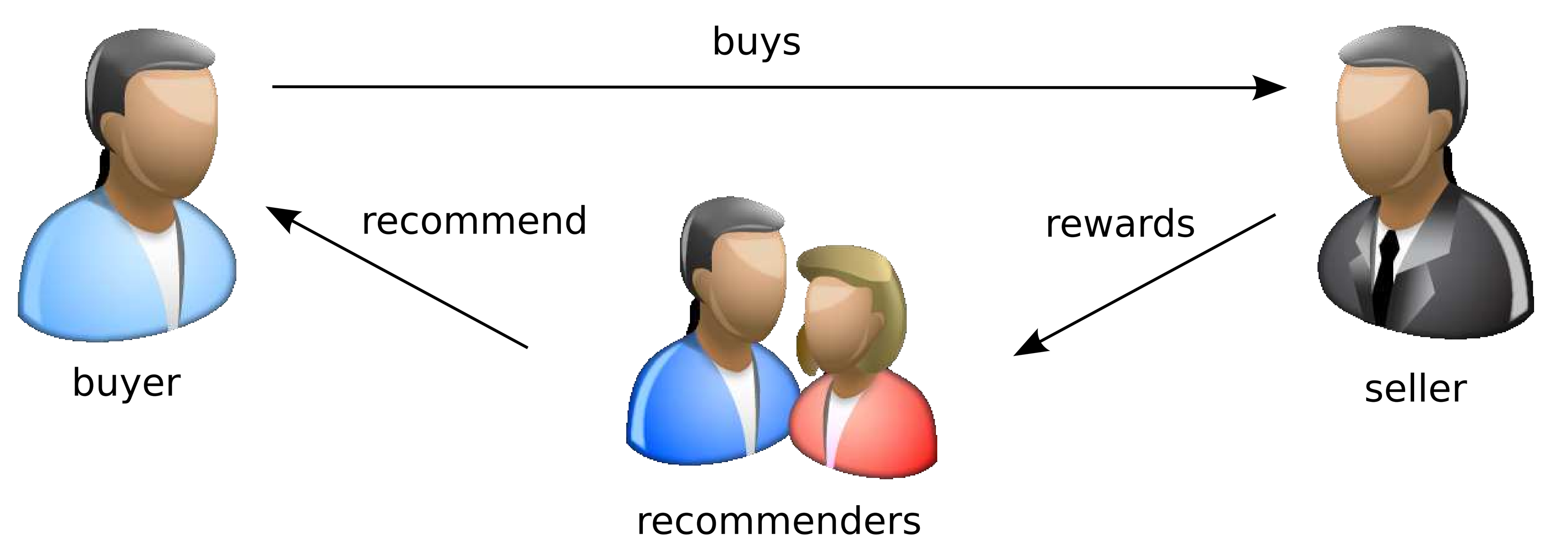
Paul Dütting[1], Monika Henzinger[2], and Ingmar Weber[3]

[1]Ecole Polytechnique Fédérale de Lausanne, Switzerland
[2]University of Vienna, Austria
[3]Yahoo! Research Barcelona, Spain

paul.duetting@epfl.ch

Problem

You recommend a product to your friend. He buys it. How much should the seller pay you?



Basic Model

Input:

- Seller $s = 0$, recommenders $R = \{1, ..., n\}$
- Margin δ (given by seller)
- For each subset $S \subseteq N = \{0, \ldots, n\}$ a value $v(S)$:
  $v(S) = \mathbf{Pr}[\text{purchase when recommended by } i \in S \setminus \{s\}] \cdot \delta$

  [Assumption: $v(S) = 0$ if $s \notin S$]

Output:

- Payoff vector $x = (x_0, \ldots, x_n)$ such that $x_i \geq 0$ and $\sum_i x_i = v(N)$

Results

1. No pricing scheme with $\sum_{i>0} x_i > 0$ is **truthful** for the seller.
2. Pricing scheme using Shapley value is **fair**.

Shapley Value

The Shapley value $\phi(v) = (\phi_0(v), \ldots, \phi_n(v))$ is given by $\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(|N|-1-|S|)!}{|N|!} \cdot (v(S \cup \{i\}) - v(S))$.

Example

Seller $s = 0$, recommenders $R = \{1, 2\}$

Non-zero values:
$v(\{0\}) = 0.1 \cdot \delta$
$v(\{0, 1\}) = (0.1 + 0.1) \cdot \delta$
$v(\{0, 2\}) = (0.1 + 0.5) \cdot \delta$
$v(\{0, 1, 2\}) = (0.1 + 0.1 + 0.5) \cdot \delta$

Shapley value:
$\phi_0(v) = (0.1 + (0.1 + 0.5)/2) \cdot \delta = 0.4 \cdot \delta$
$\phi_1(v) = 0.2/2 \cdot \delta = 0.05 \cdot \delta$
$\phi_2(v) = 0.5/2 \cdot \delta = 0.25 \cdot \delta$

Extended Model

Input:

- Seller $s = 0$, recommenders $R = \{1, ..., n\}$
- Margin δ (given by seller)
- Arguments $A = \{a_1, ..., a_k\}$, arguments $B_i \subseteq A$ used by $i \in R$ (given by recommender)
- For each subset $B \subseteq A$ a value $v(B)$:
  $v(B) = \mathbf{Pr}[\text{purchase when arguments in } B \text{ are used}] \cdot \delta$

Output:

- Payoff vector $x = (x_0, \ldots, x_n)$ such that $x_i \geq 0$, $x_i = 0$ if $i \in R$ and $B_i = \emptyset$, and $\sum_{i>0} x_i = v(\cup_i B_i)$

Results

1. No pricing scheme with $\sum_{i>0} x_i > 0$ is **truthful** for the seller.
2. Pricing scheme using Shapley value is **fair**, but **not truthful** for the recommenders.
3. Pricing scheme using anonymity-proof Shapley value is **fair**, and **truthful** for the recommenders.

Anonymity-Proof Shapley Value

For any set $B = \cup B_i$ of declared arguments the anonymity-proof Shapley value $\psi_a(v)$ for $a \in B$ is: $\psi_a(v) = \frac{\phi_a(v)}{\sum_{b \in B} \phi_b(v)} \cdot v(B)$.

Example

Seller $s = 0$, recommenders $R = \{1, 2\}$

Arguments $A = \{a_1, a_2, a_2\}$

Non-zero values:
$v(\{a_1, a_2\}) = v(\{a_1, a_3\}) = v(\{a_1, a_2, a_3\}) = \delta$

True arguments: $B_1 = \{a_1\}$ and $B_2 = \{a_1, a_2\}$

Shapley value:
$\phi_{a_1}(v) = 1/2 \cdot \delta$ and $\phi_{a_2}(v) + \phi_{a_3}(v) = 1/3 \cdot \delta$

Anonymity-proof Shapley value:
$\psi_{a_1}(v) = 3/5 \cdot \delta$ and $\psi_{a_2}(v) + \psi_{a_3}(v) = 2/5 \cdot \delta$

Falsified arguments: $B_1 = \{a_1\}$ and $B_2 = \{a_2\}$

Shapley value:
$\phi_{a_1}(v) = 1/2 \cdot \delta$ and $\phi_{a_2}(v) = 1/2 \cdot \delta > 2/5 \cdot \delta$

Liar is better off ↯

Anonymity-proof Shapley value:
$\psi_{a_1}(v) = 3/4 \cdot \delta$ and $\psi_{a_2}(v) = 1/4 \cdot \delta < 2/5 \cdot \delta$

Liar is worse off ✓

References:

P. Dütting, M. Henzinger, and I. Weber. On the pricing of recommendations and recommending strategically. Working paper (available through arXiv), 2009.
N. Ohta, V. Conitzer, Y. Satoh, A. Iwasaki, and M. Yokoo. Anonymity-proof shapley value. Autonomous Agents and Multiagent Systems, pp. 927–934, 2008.