Simplicity-Expressiveness Tradeoffs in Mechanisms

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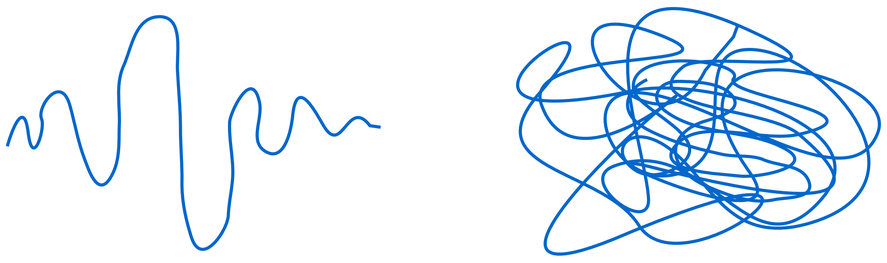
"Everything should be made as simple as possible, but no simpler." – Albert Einstein

Simplicity vs. Expressiveness



Fully expressive mechanisms might be infeasible, because of the sheer number of possible outcomes. Simplified mechanisms alleviate this problem.

Depending on the information held by the agents simplification can preclude undesirable equilibria or facilitate coordination.

Whether or not simplification causes a loss in efficiency depends on the agents' knowledge about the types of the other agents.

Framework

Mechanism and Simplified Mechanism

A mechanism $\boldsymbol{M}$ is defined by a set of agents $\boldsymbol{N}$, a message space $\boldsymbol{X}$, a social choice function $\boldsymbol{f}$, and a payment function $\boldsymbol{p}$.

A simplified mechanism $\hat{\boldsymbol{M}}$ restricts the message space of the mechanism $\boldsymbol{M}$ it is derived from to $\hat{\boldsymbol{X}} \subseteq \boldsymbol{X}$, but is otherwise identical to $\boldsymbol{M}$.

Complete Information vs. Incomplete Information

With complete information agents are assumed to know each other types and are allowed to choose their strategies $\boldsymbol{s_i}$ based on this information.

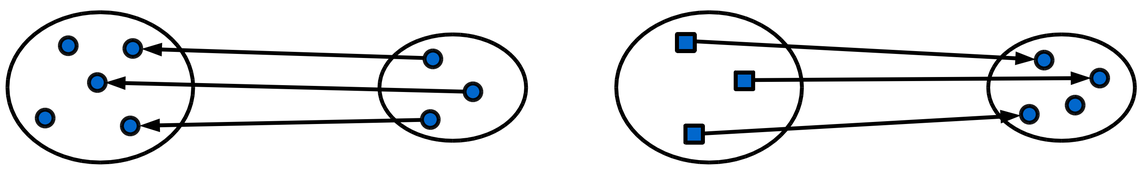
With (strict) incomplete information agents have no knowledge about the other agents' types. An agent's strategy $\boldsymbol{s_i}$ may be based on his type.

Solution Concepts

With complete information / (strict) incomplete information a profile of strategies $\boldsymbol{s}$ is a Nash equilibrium / ex post equilibrium if each agent's strategy is a best response to the other agents' strategies.

Properties of Simplified Mechanisms

Tightness and Totality



A simplified mechanism is tight if does not introduce new equilibria.

A simplified mechanism is total if it leaves all equilibria of the original mechanism intact.

Positive Revenue and Vickrey Compatibility

A simplified mechanism has positive revenue if in every equilibrium its revenue is strictly larger than zero.

A simplified mechanism is Vickrey compatible if in some equilibrium its outcome coincides with the outcome of VCG for truthful bids.

Complete Information

Sponsored Search / Assignment Problem

GSP and VCG always have a zero revenue equilibrium. This equilibrium is efficient, gives higher utilities, and is unique under costly bidding. [Milgrom]

16	**12**	**8**	**@8**		**16**	**0**	**0**	**@0**
8	**6**	**4**	**@2**	$\rightarrow$	**0**	**6**	**0**	**@0**
4	**3**	**2**	**@0**		**0**	**0**	**2**	**@0**

Simplifications $\boldsymbol{\alpha}$-GSP and $\boldsymbol{\alpha}$-VCG of GSP and VCG that ask each agent for a single bid $\boldsymbol{b_i}$ and derive his on slot $\boldsymbol{j}$ by multiplying it with click trough rate $\boldsymbol{\alpha_j}$ are tight, have positive revenue, and are Vickrey compatible. [Milgrom]

We show that for both $\boldsymbol{\alpha}$-GSP and $\boldsymbol{\alpha}$-VCG and every $\boldsymbol{\epsilon > 0}$ there exist instances with revenue $\leq \boldsymbol{\epsilon}$. That is, overall revenue can be arbitrarily small.

We show that for arbitrary valuations there is a tight and Vickrey compatible simplification of GSP (namely $\boldsymbol{\alpha}$-GSP with $\boldsymbol{\alpha = (1, \ldots, 1)}$) that has considerable revenue on almost all instances, while any simplification of VCG with these properties has arbitrarily low revenue on all instances.

Combinatorial Auctions

The zero revenue problem continues to exist. There exist simplifications that are tight, have positive revenue, and are Vickrey compatible. [Milgrom]

We show that there exists a tight and total (= fully expressive) simplification of VCG which requires every agent to submit up to $\boldsymbol{n}$ numbers. ($\neq$ [Benisch et al.])

Incomplete Information

Sponsored Search / Assignment Problem

We show that truthful bidding is the unique efficient equilibrium of $\boldsymbol{\alpha}$-VCG and, thus, zero revenue is not a problem.

We show that for arbitrary valuations a tight simplification of VCG with less than $\boldsymbol{k}$ bids per agent is to partition the items into bundles, ask agents for a bid on each bundle, but give them only one of the items in the bundle in case they win. But this is generally not efficient.

Combinatorial Auctions

We show that allowing the agents to bid on a subset $\boldsymbol{\Sigma}$ of bundles and deriving bids on all other bundles as the maximum over all bids on contained bundles is a tight simplification if and only if $\boldsymbol{\Sigma}$ is a quasi-field.

$\boldsymbol{\Sigma = \{\emptyset, A, AC, ABC\}}$ ↯ $\qquad$ $\boldsymbol{\Sigma = \{\emptyset, A, BC, ABC\}}$ ✓

We show that the set of ex post equilibria of this simplification is given by truthful bidding on all sub-quasi-fields $\boldsymbol{\Sigma' \subseteq \Sigma}$ and that truthful bidding on $\boldsymbol{\Sigma}$ is simultaneously best for all agents, but generally not efficient. [Holzman et al.]

Bidding on all bundles requires an exponential number of bids. Among the ex post equilibria of VCG with at most $\boldsymbol{b}$ bids there are several Pareto optimal ones, but generally none of them is simultaneously best for all of them.

Future Work

It would be interesting to quantify the tradeoff between simplicity and expressiveness, but it is not yet clear how to do it.

References:

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